11. a) Prove that eigen values of a real symmetric matrix is real.

b) Show that $\begin{bmatrix} 1 & 2+i & -1 \\ 2-i & 2 & i \\ -1 & -i & 0 \end{bmatrix}$ is a Hermitian matrix.

c) If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, then prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$ 3+2+5

12. a) Find the shortest distance between two skew lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

b) Show that the equation of the plane through the points (2, 3, –3), (1, 1, –2), (–1, 1, 4) is $3x - y + z = 0$ 5+5

13. a) Find the equation of the sphere described on the join of points P(2, –3, 4) and Q(–5, 6, 7) as diameter.

b) Find the equation of the plane passing through the points (1, 1, 2) and (2, 4, 3) and perpendicular to the plane $x - 3y + 7z + 5 = 0$ 5+5

14. a) Show that the equation of the plane passes through the point P(a, b, c) and perpendicular to the straight line OP, where O is the origin, is $ax + by + cz = a^2 + b^2 + c^2$.

b) Show that the points (3, 9, 4), (4, 5, 1), (–4, 4, 4) and (0, –1, –1) are co-planer. 5+5

Ex/CHE/Math/T/123/2018

Bachelor of Engineering in Chemical Engineering Examination, 2018

(1st Year, 2nd Semester)

Mathematics - II

Time : Three hours Full Marks : 100

(50 marks for each group)

Symbols and Notations have their usual meaning.

GROUP - A (50 MARKS)

(Answer ***any five*** questions

and bold letters indicate the vector quantity)

1. a) Find a unit vector **u** parallel to the resultant **R** of vectors $\mathbf{r}_1 = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{r}_2 = -\mathbf{j} + 6\mathbf{k}$.

b) Suppose $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{B} = 5\mathbf{i} + \mathbf{j}$, $\mathbf{C} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, Find A. $(\mathbf{B} \times \mathbf{C})$.

c) Show that the vector $\mathbf{V} = (y^2 + z^3)\mathbf{i} + (2xy - 5z)\mathbf{j} + (3xz^2 - 5y)\mathbf{k}$ irrotational.

d) Also show that **V** can be expressed as the gradient of a scalar function. 2+2+2+4

2. a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = -t - 5$. Find the components of its velocity and acceleration at t = 1 in the direction of $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

b) $\varphi(x,y,z) = x^2yz - 4xyz^2$. Find the directional derivative of φ at the point (1,3,1) in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

c) Suppose $\mathbf{A} = x^2z^2\mathbf{i} - 2y^2z^2\mathbf{j} + xy^2z\mathbf{k}$. Find *curl curl* **A** and *div curl* **A**. 3+3+4

3. a) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$.

b) Suppose $\mathbf{F} = y\mathbf{i} + (x - 2xz)\mathbf{j} - xy\mathbf{k}$. Evaluate $\iint (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, \mathbf{ds}$ over the surface S where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 3+7

4. Prove that a Necessary and Sufficient Condition that $\oint \mathbf{F} \cdot \mathbf{dr} = \mathbf{0}$ over C for every closed path C is that $\nabla \times \mathbf{F} = \mathbf{0}$ identically. 10

5. a) Evaluate by Green's Theorem,

$\oint [(\cos x \sin y - xy)dx + \sin x \cos y dy]$ over C where C is the circle $x^2 + y^2 = 1$.

b) State The Divergence Theorem of Gauss.

c) Prove $\iiint_V \text{div}\mathbf{n}dv = S$ by Gauss Divergence Theorem, where V is a region bounded by surface S. 6+2+2

b) Show that the following equations are *consistent* and solve them.

$$\begin{aligned} x - 3y - 8z + 10 &= 0 \\ 3x + y - 4z &= 0 \\ 2x + 5y + 6z - 13 &= 0 \end{aligned}$$

5+5

10. a) State *Cayley Hamilton* theorem. Using this theorem show that the matrix A satisfies the equation $A^3 - 23A - 40I = 0$. Hence find A^{-1} where,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

b) Show that

$$\begin{vmatrix} -bc & bc + b^2 & bc + c^2 \\ ca + a^2 & -ca & bc + c^2 \\ ab + a^2 & ab + b^2 & -ab \end{vmatrix} = (bc + ca + ab)^3$$

c) Determine the eigen values of the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

[(2+3)+3+2]

6. a) Evaluate $\oint \mathbf{F}\cdot\mathbf{dr}$ over C by Stokes' theorem, where $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j} - (x+z)\mathbf{k}$ and C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).

b) Evaluate $\iint \mathbf{F}\cdot\mathbf{dr}$ over S where $\mathbf{F} = x\mathbf{i} - y\mathbf{j} + (z^2-1)\mathbf{k}$ and S is the closed surface bounded by the planes z = 0, z = 1 and the cylinder $x^2 + y^2 = 4$ by the application of Gauss Divergence Theorem. 5+5

7. If φ and ψ are scalar point functions which together with their derivative in any direction are uniform and continuous within the region V bounded by a closed surface S. Then prove that

a) $\iiint_V [\varphi d'^2\psi + d'\varphi\cdot d'\psi]dv = \iint_S (\varphi d'\psi)\cdot\mathbf{ds}$

b) $\iiint_V [\varphi d'^2\psi - \psi d'^2\varphi]dv = \iint_S (\varphi d'\psi - \psi d'\varphi)\cdot\mathbf{ds}$ 5+5

GROUP - B

(Answer ***any five*** questions.)

8. a) Show that the square matrix A can be expressed as the sum of a *symmetric* and a skew *symmetric* matrix, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$$

b) For what value of K, the matrix

$$\begin{bmatrix} 3-k & 2 & 2 \\ 2 & 4-k & 1 \\ -2 & -4 & -1-k \end{bmatrix}$$ *is singular* ?

c) Prove that $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is a *unitary* matrix. 4+3+3

9. a) Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ using *Normal Form.*