

**BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING
EXAMINATION, 2018**

(1st Year, 1st Semester, Supplementary)

MATHEMATICS - I

Time : Three hours

Full Marks : 100

Answer **any (10) ten** questions.

11. a) Show that $\int_0^1 \sqrt{1-x^4} dx = \{\Gamma(1/4)\}^2 / 6\sqrt{2}\pi$

b) Find $\int_0^{\pi/2} \sqrt{\tan x} dx$ 5+5

12. a) Find the solid formed by the rotation by a ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

b) Find the volume of the solid generated by revolving the cardioide $r = a(1 - \cos \theta)$ about the initial line.

1. a) $\frac{x+b}{2} = a \tan^{-1}(a \log_e y), a > 0$

Prove $yy'' - yy' \log y = (y')^2$.

b) If $f\left(\frac{x+y}{2}\right) = f(x) + f(y)/2, f'(0) = a, f(0) = b$, then
find $f''(x)$, where y is independent of x . 4+6

2. Find $y_n(0)$ when $y = \log(x + \sqrt{1+x^2})$ 10

3. a) i) If $f(x) = 0$, for all values x in a interval then $f(x)$ is constant in that interval

ii) If $\phi'(x) = \psi'(x)$ in an interval then $\phi(x)$ and $\psi(x)$ is differ by a constant in that interval.

b) If $f(h) \pm f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$ where $0 < \theta < 1$,

find θ , when $h = 1$ and $f(x) = (1-x)^{5/2}$ 4+6

4. a) Expand $(\sin^{-1}x)^2$ in ascending power of x .
 b) Expand $\tan x$ in power of x as far as x^5 . 6+4
5. Let $f(x) = (1+x)^m$, when m is any real number. Expand it by using Maclaurin's series. 10
6. a) Show that

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$$

b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$ 4+6

7. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that

i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 3/(x + y + z)$

ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -3/(x + y + z)^2$ 10

8. a) If $u = \tan^{-1} \left(\frac{x^2 + y^2}{x - y} \right)$ show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

- b) Verify Euler theorem for the following function

$$u = \sin \frac{x^2 + y^2}{xy} \quad 5+5$$

9. Evaluate :

a) $\int_0^{\alpha} \frac{\log \left(x + \frac{1}{x} \right)}{1 + x^2} dx$

b) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ if it converges. 10

10. Evaluate

i) $\int_0^{\alpha} e^{-x} \cdot x^{3/2} dx$

ii) $\int_0^{\alpha} \sqrt{x} \cdot e^{-x^3} dx$

iii) $\int_0^{\alpha} \sqrt{x^n} \cdot e^{-ax^n} dx$

Where m, n and a are +ve integer. 10