13. a) Evaluate

$$\iint_R \sin(x+y)\,dxdy \text{ over } R:\{0 \le x \le \pi/2,\ 0 \le y \le \pi/2\}$$

b) Find the area of the loop of the curve

$$ay^2 = x^2(a-x)$$ 3+7

14. a) Employ Simpson's $\frac{1}{3}$ rule to evaluate $\int_0^1 \frac{dx}{1+x^2}$ by dividing the interval 0 to 1 into four equal points.

b) Find the volume of a sphere of reaius a. 6+4

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester)

MATHEMATICS - I

Time : Three Hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five (5)*** questions

1. a) If $y = \frac{1}{x^2+a^2}$, then show that

$$y_n = \frac{(-1)^n \cdot n!}{a^{n+2}} \text{Sin}^{n+1}\theta \text{Sin}(n+1)\theta$$

where $\theta = \cot^{-1}\frac{x}{a}$.

b) If $x = \sin t,\ y = \sin kt$, when k is constant show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$$ 6+4

2. If $y = \cos(m\sin^{-1} x)$. Show that

i) $(1-x^2)y_2 - xy_1 + m^2 y = 0$

ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y = 0$

Also find the value of y_n when $x = 0$. 10

3. State and prove mean value Theorem. Give a geometrical interpretation of the theorem. 10

4. Expand $f(x) = \log(1+x)$ in powr of x in infinite series stating the condition under which the expansion is valid. 10

5. Evaluate :

a) $\underset{x\to 0}{\text{Lt}} \left(\frac{\tan x}{x}\right)^{1/x}$

b) Find a, b such that

$$\underset{x\to 0}{\text{Lt}} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$ 5+5

6. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, prove that

a) $x\frac{\partial u}{dx} + y\frac{\partial u}{\partial y} = \sin 2u$

b) $x^2\frac{\partial^2 u}{dx^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{dy^2} = (1-4\sin^2 u)\sin 2u$ 4+6

10. a) Show that the integral $\int_a^b \frac{dx}{(x-a)}\mu$ is convergent if and only if $\mu < 1$.

b) Show that $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}}dx$ is convergent.

6+4

11. a) Examine the convergence of the improper integral $\int_0^\alpha \frac{x^{p-1}}{1+x}dx$.

b) Show that the improper integral is $\int_0^\alpha \frac{\sin x}{x}dx$ convergent.

5+5

12. a) Prove that

$$\int_0^{\pi/2} \sin^p x dx \times \int_0^{\pi/2} \sin^{p+1} x dx = \frac{\pi}{2(p+1)},\ p > -1.$$

b) Prove that

$$\int_a^b (x-a)^{m-1}(b-x)^{n-1}dx = (b-a)^{m+n-1}.\beta(m,n),$$

$m > 0,\ n > 0$ 5+5

[Turn over

PART - II

Answer ***any five (5)*** questions

8. a) Prove that the function f defined on [a, b] by $f(x) = x,\ x \in [a,b]$ is Riemann integrable on [a, b] showing $\int_{-a}^{b} f = \int_{a}^{-b} f$. Evaluate $\int_{a}^{b} f$.

   b) Show that f, defined by $f(x) = \operatorname{sgn} x,\ x \in [-2,2]$, is integrable on [–2, 2] 7+3

9. a) Let f be defined on [–2, 2] by

$$f(x) = 3x^2 \cos\frac{\pi}{x^2} + 2\pi \sin\frac{\pi}{x^2}, \quad x \neq 0.$$
$$= 0 \qquad x = 0$$

   Show that f is integrable on [–2, 2].

   Evaluate $\int_{-2}^{2} f$.

   b) If $f : [a,b] \to \mathbb{R}$ be continuous on [a, b] and $\int_{a}^{b} f(x)dx = 0$, prove that there exists at least a point $c \in [a,b]$ such that $f(c) = 0$

   c) State second Mean Value Theorem in Bonnet's form.

5+3+2

7. a) Find the extreme value of

$$f(x,y) = 2x^2 - xy + 2y^2 - 20x$$

   b) Test the convergence of the series

$$\frac{x}{1^2} + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \cdots$$ 6+4