13. (a) A simple random sample of size 5 is drawn without replacement from a finite population consisting of 41 units. If the population standard deviation is 6.25, what is the standard error of sample mean.

(b) A population consists of the four members 3, 7, 11,15. Consider all possible samples of size two which can be drawn with replacement from this population. Find the population mean and population standard deviation. 8+8

14. Consider the differential equation

M(x,y)dx + N(x,y)dy = 0

where M and N have continuous first order partial derivatives at all points (x,y) in a domain D. If the above differential equation is exact in D, then whow that

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ for all (x,y) in D. 2

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Ex./ARCH/MATH/T/216/4/2018(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2018

(2nd Year, 1st Semester, Old Syllabus)

Mathematics - III A

Time : Three hours Full Marks : 100

The figures in the margin indicate full marks.
All Symbols/notations have their usual meaning
Use a separate Answer-Script for each part.

PART - I

Answer any ***five*** questions.

1. (a) State and prove law of addition of probability for any two events. 1+5

(b) What is the probability that all 3 children in a family have different birthdays ? (assuming, 1 year = 365 days). 4

2. (a) If A and B are two independent events corresponding to a random experiments then show that

(i) A and B^c and

(ii) A^c and B^c are also independent. 3+3

(b) A four-digit number is formed by the digits 1, 2, 3, 4 with no repetition. Find the probability that the number is
(i) odd
(ii) divisible by 4. 2+2

3. (a) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys. One child is selected at random from each group. Find the probability that the selected group consists of 1 girl and 2 boys. 6

(b) If 20 dates are named at random, what is the probability that 3 of them will be Sundays ? 4

4. (a) Find the mean and variance of the Binomial distribution. State the condition for maximum variance. 5

(b) A point chosen at random in a given interval divides it into two subintervals. Find the probability that the ratio of the length of the left sub-interval to that of the right sub-interval is less than a constant K. 5

5. (a) State the essential properties of a spherical triangle. 3

(b) Define the following : 3

(i) Zenith, (ii) Celestial Pole, (iii) Celestial equator.

11. (a) Find the family of curves for which the angle between the radius vector and the tangent at (r, θ) is one half of the vectorial angle.

(b) Find the curve for which cartesian subtangent is constant.

(c) Find the orthogonal trajectories of the family of hypocycloids $x^{2/3} + y^{2/3} = a^{2/3}$, where 'a' is a variable parameter.

(d) A body of mass 'm' falls vertically by the action of gravity. It experiences an atmospheric resistance KU^2 when its velocity is ν (k = constant). Show that the distance travelled by the body in time 't' is given by

$$x = \frac{a}{b} \log \cos h(bt)$$

with,

$$a = \sqrt{\frac{mg}{k}} \text{ and } b = \frac{ak}{m}$$

4+3+4+5

12. Solve the following differential equations :

(a) $(D^2 + 4d + 4)y = \sin h\ 2x$

(b) $(D+2)\,(D-1)^3 y = e^x$. 8+8

(b) The normal at any point P of a curve cuts OX at G and N is the foot of the ordinate of P. If NG varies as the square of the radius vector from O, find the family of curves, each member of which satisfies the property stated above. 8+8

9. (a) Solve the linear differential equation :

$$\frac{dy}{dx} + \frac{n}{x}y = \frac{9}{x^n}$$

where n and a are constants.

(b) Show that the differential equation

$ydx + (x^2y - x)dy = 0$

is not exact. Find an integrating factor and hence solve the differential equation.

(c) Solve the Bernoulli's equation

$$\frac{dy}{dx} - \frac{2y}{2x} = x^2 e^x, \; y(1) = 0$$ 4+6+6

10. (a) Solve : $(D^2 - 3D + 2)y = \sin 3x$, $D = \frac{d}{dx}$ 8

(b) Apply the method of variation of parameters to solve :

$$y^2 + a^2y = \text{cosec } ax$$ 8

(c) If the declination and Zenith distance of a star on the prime vertical are known then what is the latitude of the place of the observation ? 4

6. (a) Define horizontal and equatorial co-ordinate system. Which co-ordinate system is preferable to identify a heavenly body ? 5

(b) If h and $h^{/}$ are the hour angles of a star of declination δ on the prime vertical and at setting respectively, show that $\cos h \cos h^{/} + \tan^2 \delta = 0$. 5

7. (a) In spherical trigonometry state the cosine and four parts formula. 4

(b) State Sun's co-ordinates in equatorial and ecliptic system on summer and winter solstices, vernal and autumnal equinoxes. 6

PART - II

Answer ***q.no. 14*** and any ***three*** from the rest. 2+3x16

8. (a) Solve : $(x\cos x)\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

(Turn over)