Ex./ARCH/MATH/T/124/2018

BACHELOR OF ARCHITECTURE ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - II

Time : Three hours   Full Marks : 100

Use a separate Answer-Script for each part.

PART - I (40 marks)

Answer any ***four*** questions.

1. (a) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(b) Solve the equations

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

5+5

2. (a) Solve by Cramer's rule

$3x+y+2z=3;\ \ 2x-3y-z=-3;\ \ x+2y+z=4$

(Turn Over)

(b) Express

$$\begin{vmatrix} (2bc-a^2) & c^2 & b^2 \\ c^2 & (2ca-b^2) & a^2 \\ b^2 & a^2 & (2ab-c^2) \end{vmatrix}$$

as a square of a determinant and hence find its value. 5+5

3. (a) If $A+B=\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A-B=\begin{bmatrix} 3 & 1 \\ 1 & A \end{bmatrix}$, Calculate the product AB.

(b) Prove that $A^3-4A^2-3A+11I=0$, where

$$A=\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$ 4+6

4. (a) Find the inverse of $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and hence show that $AA^{-1}=I$.

11. (a) Find the point where the straight line joining the points (2,–3,1) and (3,–4,–5) cuts the plane 3x+y+z=8.

(b) Find the equation of the straight line through the point (α,β,γ) at right angles to the st.lines $\frac{x}{l_1}=\frac{y}{m_1}=\frac{z}{n_1}$ and $\frac{x}{l_2}=\frac{y}{m_2}=\frac{z}{n_2}$. 5+5

12. Find the equation of the straight line in symmetrical form whose general equation is given by x–2y+3z=4, 2x–3y+4z = 5. Also find its direction cosines. 10

13. (a) Find the image of the point (–3,8,4) in the plane 6x–3y– 2z +1=0.

(b) Find the condition that the straight line $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$; $\frac{x}{a\alpha}=\frac{y}{b\beta}=\frac{z}{c\gamma}$ and $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ will lie on a plane. 3+7

14. Find the shortest distance between the straight lines $\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1}$ and $\frac{x+3}{3}=\frac{y+7}{-2}=\frac{z-6}{-4}$. Also find the equation of the line of shortest distance. 10

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7. (a) Determine the coordinate of the centre and lengths of the semi axes of the ellipse $9x^2+4y^2+18x-16y=11$.

   (b) Find the equation of the hyperbola whose eccentricity is $\sqrt{3}$, one of whose foci is (3,2) and corresponding directrix is $2x+y=1$. 5+5

8. (a) Find the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

   (b) If the equation of one asymptotes of the hyperbola $9x^2-16y^2+90x+32y=367$ be $3x-4y+19=0$, then find the other asymptotes. 4+6

9. (a) Find the ratio in which the line segment joining the points (2,–3,5) and (7,1,3) is divided by the xy-plane.

   (b) Find the projection of the line segment joining the points (3,3,5) and (5,4,3) on the straight line joining the points (2,–1,4) and (0,1,5). 5+5

10. (a) Find the equation of the plane which passes through the point (2,1,–1) and is orthogonal to each of the planes $x-y+z=1$ and $3x+4y-2z=0$.

    (b) Find the equation of the plane passing through the intersection of the planes $2x+y+2z=9$ and $4x-5y-4z=1$ and the point (3,2,–1). 6+4

   (b) Find the rank of

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

6+4

5. (a) Write the 'Rouche's theorem' related to consistency and inconsistency of system of linear equation.

   (b) Investigate the values a and b for which the equations $x+ay+z=3$, $x+2y+2z=b$, $x+5y+3z=9$ are consistent. When these equations have a unique solution. 2+8

PART - II (60 marks)

Answer any ***six*** questions.

6. (a) Find the tangent to the curve

$$\frac{x^2}{4}-\frac{y^2}{9}=1\, at\, (a,b)$$

   (b) Find the curvature of the curve $x=a(t-\sin t)$, $y=a(1-\cos t)$ at $t=\pi$. 5+5