BACHELOR OF ARCHITECTURE EXAMINATION, 2018 (OLD)

(1st Year, 2nd Semester, Old)

MATHEMATICS – II A

Time : Three hours Full Marks : 100

Answer any ***TEN*** questions.
(Notations have their usual meanings)

1. (a) Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2.$$

(b) Find the value of x, which satisfy the equation

$$\begin{vmatrix} a-x & c & c \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0, \text{ when } a+b+c=0. \qquad 5+5$$

2. (a) Show that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^2$$

(b) Find the solution of the following system of equation by Cramer's rule

$$x + 2y + z = 1$$

$$3x + y + 2z = 3$$

$$x + 7y + 2z = 1. \qquad 5+5$$

3. (a) Verify that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix. Hence find A^{-1}.

(b) Find the solution of the following system of equation by matrix method

$$x + y + z = 4$$

$$2x - y + 3z = 1$$

$$3x + 2y - z = 1. \qquad 5+5$$

4. (a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, show that $A^3 - A = A^2 - I$ and hence find A^{-1}.

(b) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, prove that $A^3 = A^{-1}$. 5+5

5. (a) Show that

$$\begin{vmatrix} a^2 & bc & c^2 + ca \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

(b) Find the value of x, which satisfy the equation

$$\begin{vmatrix} x^3 - a^3 & x^2 & x \\ b^3 - a^3 & b^2 & b \\ c^3 - a^3 & c^2 & c \end{vmatrix} = 0.$$ 5+5

6. (a) Find the rank of the matrix

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}.$$

(b) Find the adjugate and reciprocal determinant of

$$\begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}$$

5+5

7. Find the maximum value of $z = 2x_1 + x_2$

Subject to the constraints:

$4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$4x_1 - x_2 \leq 8$

$x_1, x_2 \geq 0.$ 10

8. (a) Obtain the equation of the cylinder whose generators intersect the ellipse $9x^2+3y^2 = 1$; $z = 0$ and are parallel to the straight line with direction ratios are $(1,-1,1)$.

(b) Find the equations of the planes through the straight line

$2x - y + 3z + 2 = 0 = 3x + 2y - z + 3$

Parallel to the coordinates axes. 5+5

9. (a) Find the equation of the plane passing through the three points (2, 2, -1), (3, 4, 2) and (7, 0, 6).

(b) Find the points where the straight line joining the points (2, -3, 1) and (3, -4, -5) cuts the plane $3x + y + z = 8$. 5+5

10 (a) Find the distance of the point (4, 1, 1) from the straight line

$x + y + z = 4$, $x - 2y - z = 4$.

(b) Find the value of c for which the plane $x + y + z = c$ touches the sphere

$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$. 5+5

11. (a) Show that the planes $2x - y - z = -3$, $x + 2y + 3z = 2$ and $-2x + 4y + 2z = 4$ intersect in a straight line.

(b) Find the equation to the common tangent of the circle $x^2 + y^2 = 4ax$ and the parabola $y^2 = 4ax$. 5+5

12. (a) Find the equation of the ellipse one of whose foci is (-1, 1), eccentricity is ½ and the corresponding directrix is $y = x + 3$.

(b) Find the equations of the tangents to the circle $x^2 + y^2 + 8x + 10y - 4 = 0$ which are parallel to the straight line $x + 2y + 3 = 0$. 5+5