Ex./ARC/MATH/T/114/2018

BACHELOR OF ARCHITECTURE EXAMINATION, 2018

(1st Year, 1st Semester)

Mathematics - I

Time : Three hours | Full Marks : 100

Answer any ***ten*** questions.
All questions carry equal marks.
(Symbols/notations have their usual meaning)

1. (a) Find the expression for nth order derivative of the function $(\operatorname{Sin} x)$.

   (b) State Leibnitz's theorem.

   (c) If $x = \tan(\log y)$, then find the value of $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1}$. 3+2+5=10

2. (a) State Rolle's Mean Value Theorem.

   (b) Give an example to verify : "If differentiability fails at an interior point of the interval, the conclusion of Rolle's theorem may not hold."

   (c) Suppose $f(x)$ is continuous and differentiable on [6,15], such that $f(6) = -2$ and $f'(x) \leq 10$ for all x t (6,15). What is the largest possible value of f(15).

   2+3+5=10

(Turn over)

3. (a) Find first 3 non zero terms of Taylor's expansion for the function sin πx centred at $x = \frac{1}{2}$. Hence approximate the value of $\mathrm{Sin}\left(\frac{\pi}{2} + \frac{\pi}{10}\right)$. 4+2

(b) Find Maclaurin's series for $\mathrm{Sin}^2 x$. 4

4. (a) Compute :

(i) $\lim_{x \to \infty}\left(\sqrt{x^2 + 3x + 7 - x}\right)$ 3

(ii) $\lim_{x \to -\infty} x e^x$ 3

(b) Is y = | x | is monotonic on interval [–2, 2] ? 4

5. (a) Define : Stationery points, points of inflexion. 2+2

(b) Find the maximum value of $2x^3 - 24x + 107$ in the interval [– 3, –1] and minimum value in the interval [1,3]. 6

6. (a) Find all the 2nd order partial derivatives of the function $f(x, y) = e^{x y^2}$. 3

(b) Define homogeneous function for two variables. 2

(b) Evaluate $\iiint_E 2x \; dV$, where E is the region under the plane $2x + 3y + z = 6$, that lies in the first octant. 3+7

14. Using Simpson's rule evaluate (taking 4 sub-intervals) :

(a) $\int_0^1 \cos(x^3 + x)dx$

(b) $\int_1^3 x^z dx$ 6+4

—— X ——

10. (a) Find the area between $f(x) = -x^2 + 4x$ and $g(x) = x^2 - 6x + 5$.

(b) Determine the length of $x = \frac{2}{3}(y-1)^{3/2}$ between $1 \le y \le 4$. 6+4

11. (a) Determine the surface area of the solid obtained by rotating $y = \sqrt[3]{x}$, $1 \le y \le 2$ about the y-axis.

(b) Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = x^{1/3}$ and $y = \frac{x}{4}$ that lies in the first quadrant about y-axis. 5+5=10

12. (a) Compute $\iint_R 6xy^2 \, dA$, where $R = [2,4] \times [1,2]$.

(b) Evaluate $\iint_D (6x^2 - 40y) dA$, D is the triangle with vertices (0,3), (1,1), (5,3). 3+7

13. (a) Evaluate $\iint_B 8xyz \, dV$, where $B = [2,3] \times [1,2] \times [0,1]$.

(c) State Euler's theorem for homogeneous function. 2

(d) Find the value of $x\frac{\partial t}{\partial x} + y\frac{\partial t}{\partial y}$ for $\frac{x^2 + y^2}{x^5 - y^5}$. 3

7. (a) Integrate : $\int \frac{x^2 + x - 1}{x(x^2 - 1)} dx$.

(b) Using definition of definite integral evaluate,

$$\int_0^4 \left(-3x^2 + 5x - 1\right) dx$$ 4+=6=10

8. Check the convergence of the followings :

(a) $\int_1^\infty \frac{\sin x}{x^3} dx$ (b) $\int_1^\infty \frac{dx}{\sqrt{x+1}}$ 5+5

9. Evaluate the following integrals :

(a) $\int_0^\infty \sqrt{x} \, . \, e^{-\sqrt[3]{x}} dx$

(b) $\int_0^{\pi/2} \sin^6 \theta \, d\theta$ 5+5