M.E. PRODUCTION ENGINEERING 1st YEAR 1st SEMESTER, 2018

THEORY OF OPTIMIZATION

Time: **Three hours** Full marks: **100**

Answer any *THREE questions* from *GROUP A (PART II)* and any *TWO questions* from *GROUP B (Part I)*

GROUP A / PART II

1.(a) Player A is paid Rs. 8 if two coins turn heads at the same time and Rs. 10 if two coins turn tails at the same time. Player B is paid Rs. 3 when the two coins do not match. Develop the corresponding pay-off matrix and solve the game. (8)

(b) A soft drink company calculated the market share of two of its products against its major competitor, which has three products. The company found out the impact of additional advertisement in any one of its products against the other. What is the best strategy for the company as well as the competitor? (Use graphical method) (12)

Company A	Company B		
	B1	B2	B3
A_1	6	7	15
A_2	20	12	10

2.(a) State the need for multi-objective optimization. (4)

(b) Consider a certain community in a well-defined area with three types of grocery stores; for simplicity we shall call them I, II and III. Within this community, there exists a shift of customers from one grocery store to another. A study was made on January 1st and it was found that 1/4 shopped at store I, 1/3 at store II and 5/12 at store III. Each month store I retains 90 percent of its customers and 10 percent of them to store II. Store II retains 95 percent of its customers and losses 5 percent of them to store III. Store III retains 40 percent of its customers and losses 50 percent of them to store I and 10 percent to store II. (16)

(i) What proportion of customers would each store retain by February 1st and March 1st?

(ii) Assuming that the same pattern continues, what would be the long-run distribution of customers among the three stores?

3. Minimize $Z = 4x_1^2 + 7x_2^2$ subject to $x_1 + x_2 = 5$ using the following methods: (4x5)
   (i) Direct substitution,
   (ii) Lagrangian multiplier,
   (iii) Penalty function and
   (iv) Constrained variation.

4.(a) Solve the following problems by geometric programming:

Minimize $Z = 2x_1^3x_2^{-3} + 4x_1^{-2}x_2 + 10x_1x_2 + 8x_1x_2^{-1}$, $x_1, x_2 \geq 0$ (10)

(b) Find the optimal solution of the following constrained multivariable problem using direct substitution method:

Minimize $Z = x_1^2 + (x_2 + 1)^2 + (x_3 - 1)^2$

Subject to the constraints; (i) $x_1 + 5x_2 - 3x_3 = 6$ (iii) $x_1, x_2, x_3 \geq 0$ (10)

5.(a) Use the Kuhn-Tucker conditions to solve the following non-linear programming problem: (10)

Maximize $Z = 7x_1^2 - 6x_1 + 5x_2^2$

Subject to the constraints

(i) $2x_1 + 3x_2 \leq 6$, (ii) $2x_1 + x_2 \leq 4$ and (iii) $x_1, x_2 \geq 0$

(b) Solve the following problem by using the method of Lagrangian multipliers: (10)

Minimize $Z = x_1^2 + x_2^2 + x_3^2$

Subject to the constraints

(i) $x_1 + x_2 + 3x_3 = 2$ (ii) $5x_1 + 2x_2 + x_3 = 5$ (iii) $x_1, x_2, x_3 \geq 0$

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M. PROD. E. EXAMINATION, 1st Year 2018

(1ST Semester)

SUBJECT: THEORY OF OPTIMISATION (PM)

Time : Three hours **Full Marks 100**

Use Separate Answer scripts for each Group

GROUP B / PART I

6. a) Explain Unboundedness, infeasibility and Degeneracy in a LPP Solution with suitable example. 7½

b) An engineering company currently has 3 plants (1, 2 and 3) each with the capacity to make a product which comes in 3 sizes – large, medium and small. The profit made on each large, medium and small unit is Rs.10/-, Rs.8/- and Rs.7/- respectively. The labour and machine capacities of plants 1, 2 and 3 are such that it is possible to produce 800, 650 and 450 units per week respectively. 12½

Another limiting factor in the production of each plant is the amount of storage space available. Plants 1, 2 and 3 have 1400, 1250 and 800 square metres of storage space available, respectively. And each unit of large, medium and small sizes produced per week requires 2.5, 2.0 and 1.5 square metres respectively.

The sales department has forecasted that 800, 900 and 600 of the large, medium and small sizes, respectively, can be sold per week. Also, management has decided (in order to maintain flexibility) that the relative number of units produced at the 3 plants must be in the same proportion as their labour and machine capacities.

The management wants to determine how many units of each size it should produce at three plants in order to maximise profit. **Formulate the problem as a Linear Programming Model.**

7. Maximise $Z = 16x_1 + 17x_2 + 10x_3$ 20

Subject to

$$x_1 + x_2 + 4x_3 \le 2000$$
$$2x_1 + x_2 + x_3 \le 3600$$
$$x_1 + 2x_2 + 2x_3 \le 2400$$
$$x_1 \le 30$$

Where, $x_1, x_2, x_3 \ge 0$

8. Solve the following LPP by Big M Method: 20

Maximise $Z = 5x_1 + x_2$

Subject to

$$5x_1 + 2x_2 \le 20$$
$$x_1 \ge 3$$
$$x_2 \le 5$$
$$x_1, x_2 \ge 0$$