M.E. MECHANICAL ENGINEERING EXAMINATION, 2018

(First Year First Semester)

THEORY OF ELASTICITY

Time: Three Hours **Full Marks: 100**

Parts of the same question must be answered together
Any missing data/information may be assumed with suitable justification
ANSWER ANY FOUR QUESTIONS

Q1. **[8+10+7]**

(a) Derive Cauchy's formula for determining the components of stress at any arbitrary direction.

(b) For the state of stress (in MPa), given by the following matrix, determine the principal stresses. Also find the direction cosines of any one of the principal planes.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c) Explain hydrostatic state of stress and pure shear state of stress. Show that the state of stress at any given point can be decomposed into hydrostatic and pure shear states of stress.

Q2. **[10+5+10]**

(a) Write down the differential equations of equilibrium in cylindrical coordinates. Derive any one of them.

(b) Derive the linear components of strain in rectangular Cartesian coordinates.

(c) For constant body force, show that the differential equation of equilibrium in rectangular Cartesian coordinates is given by $\nabla^2 (J_1) = 0$, where, J_1 is the first invariant of strain.

Q3. **[8+10+7]**

(a) Derive the compatibility equations in terms of strain in rectangular Cartesian coordinates. What is the physical significance of compatibility equations?

(b) For both plane stress and plane strain problems with constant body force, show that the compatibility equation in terms of stress is given by, $\nabla^2 \left(\sigma_x + \sigma_y\right) = 0$.

(c) Show that for plane elastic problems, the Airy stress function satisfies a biharmonic form of equation. Considering a suitable second degree polynomial for Airy stress function, determine the stress fields for plane elastic problems.

Q4. **[18+7]**

(a) For torsion problem of straight prismatic bars, show that the warping function ψ satisfies the equation $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$. For the same problem, show that the resultant shear stress acts tangent to the lateral boundary surface.

(b) Briefly explain plane strain problems.

Q5. **[10+15]**

(a) Considering suitable stress function, derive the stress fields for a narrow section cantilever beam loaded by a point force at the free end.

(b) Derive the general expressions of stress distribution for a thin rotating circular disk. Then find the stress distributions for a rotating disk with a central circular hole and also find the locations of occurrence of maximum stresses.

Q6. Write short notes on the following: **[5 × 5]**

(a) Lame's stress ellipsoid

(b) Stress invariants

(c) Octahedral stresses in terms of stress invariants

(d) Generalized Hooke's law

(e) Saint-Venant's principle

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