

M.E. Mechanical Engineering First Year First Semester Exam 2018
Stress and Deformation Analysis

Time: Three hours

Full Marks: 100

Answer any five questions

- 1a) Derive Cauchy's stress equation and discuss on traction vector and state of stress of a point.
 b) At a point in a stressed body the rectangular stress components are given by:

$$\begin{bmatrix} 90 & -30 & 0 \\ -30 & 120 & -30 \\ 0 & -30 & 90 \end{bmatrix} \text{ MPa}$$

Determine the principal stresses and the principal directions.

- c) Check for orthogonality of the principal stress directions.

(5+12+3)

- 2a) At a point in a stressed body the rectangular stress components are given by:

$$[\sigma] = \begin{bmatrix} 50 & 20 & 10 \\ 20 & 100 & 60 \\ 10 & 60 & 50 \end{bmatrix} \times 10^6 \text{ Pa}$$

Determine the normal and shear stresses on a plane whose outer normal has the direction cosines $l = 1/\sqrt{3}$, $m = 1/\sqrt{3}$ and $n = 1/\sqrt{3}$. Also compare the results obtained, by constructing Mohr's circles.

- b) Transform the above set of Cartesian stress components $[\sigma]$ into a new set of coordinates $Ox'y'z'$ where the new axes are defined by the following direction cosines,

	x	y	z
x'	2/3	2/3	-1/3
y'	-2/3	1/3	-2/3
z'	-1/3	2/3	2/3

(10+10)

- 3a) The displacement field for a body is given by $u = (x^2 + y^2 + 2) \times 10^{-4}$, $v = (3x + 4y^2) \times 10^{-4}$ and $w = (2x^3 + 4z) \times 10^{-4}$. What are the strain components at point (2, 2, 2) considering linear and Green's strain-displacement relationships. Also determine the strain in the direction of the line joining the point from origin.

- b) Define engineering stress, engineering strain, true stress and natural strain. A tensile specimen with 10 mm initial diameter and 60 mm gage length reaches max load of 100 kN and fractures at 75 kN. The minimum dia. at fracture is 8 mm. Determine the engineering stress at max load, true stress and strain at fracture and engineering fracture strain.

(10+4)+(2+4)

- 4a) Derive the equation $\nabla^4 \phi(x, y) = 0$ where $\phi(x, y)$ is stress function of a stressed isotropic linear elastic body under plane stress condition.

- b) Derive the planar stress fields that arise from the stress functions
 i) $\phi = ax^2 + bxy + cy^2$ and
 ii) $\phi = ax^3 + bx^2y + cxy^2 + dy^3$ ($a-d$ constants).

- c) Derive displacement equations of equilibrium for isotropic materials.

7+(2+3)+8

- 5a) Express Lamé's co-efficients λ and μ in terms of physical constants E and ν , indicating the derivation of the relations.
- b) State the failure criteria according to Tresca and Von-Mises. Sketch the above two failure criteria in 3D stress space as well as in 2D (perpendicular to the direction of σ_3).
- d) In the tension-torsion test of a thin hollow cylinder, the magnitude of stresses are $\sigma = Y/2$ and $\tau = \sqrt{3}Y/4$, where Y is the yield stress of the cylinder material. Plot the state of stress in normalized 2D principal stress plane σ/Y and τ/Y . Keeping now the normal stress at $\sigma = Y/2$, what value can the shear stress be increased to before the material yields, according to the Tresca and von Mises criterion?

6+6+8

- 6a) Define gage factor and explain the principle of strain measurement for resistance strain gages.
- b) Define bridge sensitivity and suggest any method to improve bridge sensitivity.
- c) Discuss the relative merits and demerits of null and deflection method of strain measurement.
- d) Write a short note on material of resistance strain gages
- e) How temperature effect is compensated by using dummy gauge.

4x5

- 7 a) The pressure test of a cylinder yields the following data.

Pressure in MPa	Strain reading in micro-strain	
	Gauge 1	Gauge 2
0	11380	15540
1.7	11650	15605
3.4	11920	15670
0	11380	15540

From the above data calculate the size of steel plate ($E = 207 \times 10^{11}$ KPa and $\nu = 0.3$) used in the construction of pressure vessel. The resistance strain gauges 1 and 2 are mounted on the outer surface of the vessel in tangential and axial directions respectively.

- b) For a strain field given by $\epsilon_{xx} = b \sin(\pi x/a)$, strain is measured at $x = a/2$. Determine the errors, if measurement is made using strain gauges of length $a/4$ and $a/10$ respectively.
- c) At a point on the free surface of an alloy steel machine part ($E = 200$ GPa, $\nu = 0.3$) strain readings of 700×10^{-6} , 70×10^{-6} and 500×10^{-6} were made at respective angles of 0° , 60° and 120° with respect to x-axis. Determine the magnitude and direction of the principal stress if the gage factor is 2.00

6+6+8

- 8a) State the principle of Minimum Potential Energy in structural mechanics and represent its mathematical form for stress based and displacement based formulation.
- b) Determine the critical load of a column with one end built in and the other end free, assuming the deflection curve as $y = c x^2/L^2$, where L is the length of the column and x is measured from the fixed end. Also determine its critical load by assuming the deflection curve as $y = c_1 x^2/L^2 + c_2 x^3/L^3$, and explain the improvement in result with the second assumption.

(2+2+2)+(6+8)