M.E. Mechanical Engineering First Year First Semester Exam 2018

Stress and Deformation Analysis

Time: Three hours

Full Marks: 100

Answer any five questions

1a) Derive Cauchy's stress equation and discuss on traction vector and state of stress of a point.

b) At a point in a stressed body the rectangular stress components are given by:

$$\begin{bmatrix} 90 & -30 & 0 \\ -30 & 120 & -30 \\ 0 & -30 & 90 \end{bmatrix} \text{ MPa}$$

Determine the principal stresses and the principal directions.

c) Check for orthogonality of the principal stress directions.

(5+12+3)

2a) At a point in a stressed body the rectangular stress components are given by:

$$[\sigma] = \begin{bmatrix} 50 & 20 & 10 \\ 20 & 100 & 60 \\ 10 & 60 & 50 \end{bmatrix} \times 10^6 \text{Pa}$$

Determine the normal and shear stresses on a plane whose outer normal has the direction cosines $l = 1/\sqrt{3}$, $m = 1/\sqrt{3}$ and $n = 1/\sqrt{3}$. Also compare the results obtained, by constructing Mohr's circles.

b) Transform the above set of Cartesian stress components $[\sigma]$ into a new set of coordinates Ox'y'z' where the new axes are defined by the following direction cosines,

	x	y	z
x'	2/3	2/3	-1/3
y'	-2/3	1/3	-2/3
z'	-1/3	2/3	2/3

(10+10)

3a) The displacement field for a body is given by $u = (x^2 + y^2 + 2) \times 10^{-4}$, $v = (3x + 4y^2) \times 10^{-4}$ and $w = (2x^3 + 4z) \times 10^{-4}$. What are the strain components at point (2, 2, 2) considering linear and Green's strain-displacement relationships. Also determine the strain in the direction of the line joining the point from origin.

b) Define engineering stress, engineering strain, true stress and natural strain. A tensile specimen with 10 mm initial diameter and 60 mm gage length reaches max load of 100 kN and fractures at 75 kN. The minimum dia. at fracture is 8 mm. Determine the engineering stress at max load, true stress and strain at fracture and engineering fracture strain.

(10+4)+(2+4)

4a) Derive the equation $\nabla^4 \emptyset(x, y) = 0$ where $\emptyset(x, y)$ is stress function of a stressed isotropic linear elastic body under plane stress condition.

b) Derive the planar stress fields that arise from the stress functions i) $\varphi = ax^2 + bxy + cy^2$ and ii) $\varphi = ax^3 + bx^2y + cxy^2 + dy^3$ (a-d constants).

c) Derive displacement equations of equilibrium for isotropic materials.

7+(2+3)+8

5a) Express Lame's co-efficients λ and μ in terms of physical constants E and ν, indicating the derivation of the relations.

..b) State the failure criteria according to Tresca and Von-Mises. Sketch the above two failure criteria in 3D stress space as well as in 2D (perpendicular to the direction of σ_3).

d) In the tension-torsion test of a thin hollow cylinder, the magnitude of stresses are $\sigma = Y/2$ and $\tau = \sqrt{3}Y/4$, where Y is the yield stress of the cylinder material. Plot the state of stress in normalized 2D principal stress plane σ/Y and τ/Y. Keeping now the normal stress at $\sigma = Y/2$, what value can the shear stress be increased to before the material yields, according to the Tresca and von Mises criterion?

6+6+8

6a) Define gage factor and explain the principle of strain measurement for resistance strain gages.

b) Define bridge sensitivity and suggest any method to improve bridge sensitivity.

c) Discuss the relative merits and demerits of null and deflection method of strain measurement.

d) Write a short note on material of resistance strain gauges

e) How temperature effect is compensated by using dummy gauge.

4x5

7 a) The pressure test of a cylinder yields the following data.

Pressure in MPa	Strain reading in micro-strain	
	Gauge 1	Gauge 2
0	11380	15540
1.7	11650	15605
3.4	11920	15670
0	11380	15540

From the above data calculate the size of steel plate (E = 207 x 1011 KPa and ν = 0.3) used in the construction of pressure vessel. The resistance strain gauges 1 and 2 are mounted on the outer surface of the vessel in tangential and axial directions respectively.

b) For a strain field given by $\varepsilon_{xx} = b \sin(\pi x/a)$, strain is measured at x = a/2. Determine the errors, if measurement is made using strain gauges of length a/4 and a/10 respectively.

c) At a point on the free surface of an alloy steel machine part (E = 200 GPa, ν = 0.3) strain readings of 700×10^{-6}, 70×10^{-6} and 500×10^{-6} were made at respective angles of 0°, 60° and 120° with respect to x-axis. Determine the magnitude and direction of the principal stress if the gage factor is 2.00

6+6+8

8a) State the principle of Minimum Potential Energy in structural mechanics and represent its mathematical form for stress based and displacement based formulation.

b) Determine the critical load of a column with one end built in and the other end free, assuming the deflection curve as $y = c x^2/L^2$, where L is the length of the column and x is measured from the fixed end. Also determine its critical load by assuming the deflection curve as $y = c_1 x^2/L^2 + c_2 x^3/L^3$, and explain the improvement in result with the second assumption.

(2+2+2)+(6+8)