50

Ref. No.: Ex/PG/ME/T/115B/2018

M.E. MECHANICAL ENGINEERING/MASTER OF NUCLEAR ENGINEERING/M.TECH. LASER SCIENCE AND TECHNOLOGY FIRST YEAR FIRST SEMESTER EXAM 2018

BASICS OF FINITE ELEMENT METHOD

Full Marks 100

Duration 3 hr.

Answer any five questions:

Parts of a question carry equal distribution of marks unless specified otherwise. Unless otherwise specified take the following values for different parameters (symbols have their usual meanings) $E = 200\ GPa, \nu = 0.3,\ I = 1e-8\ m^4, A = 1e-4\ m^2$. *Assume suitable values for any missing data.*

1. Answer the following questions-
   a. Derive the equilibrium equation $\nabla.\boldsymbol{\sigma} + \rho\boldsymbol{b} = \mathbf{0}$.
   b. Mention the boundary conditions and also show with mathematical derivations how the "Balance of Angular Momentum" is incorporated in the above equation.
   c. Represent the above equation using indicial notations and explain the significance of each term.
   d. Derive the final **variational statement** (weak form) for the given strong form.
2. From the differential equation of Euler-Bernoulli beam-
   $\frac{d^2}{dx^2}\left(EI\frac{d^2w(x)}{dx^2}\right) = q(x)$,
   using variational formulation show that the element stiffness matrix for beam element be-
   $$[K_e]_{Beam} = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}.$$
3. For the frame structure shown below, determine the DOF values and reactions using FE formulation.

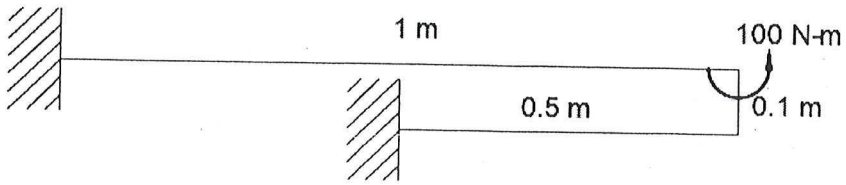

Figure (3)

4. Solve the cantilever beam problem using two CST elements, as shown in the figure, and compare stress and deflection results with closed-form solution. Thickness of the beam is 0.01 m and the elastic stiffness matrices for plane strain and plane stress cases are as following.
   $$\bar{\boldsymbol{D}}_{PlStrn} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix};\ \bar{\boldsymbol{D}}_{PlStrs} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}.$$

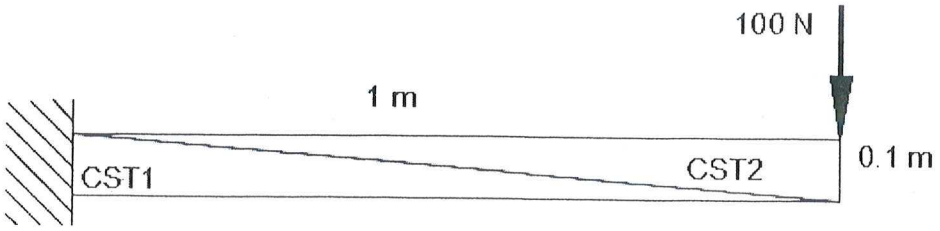

Figure (4)

5. For the truss structure shown below, determine the DOF values and reactions using FE formulation.

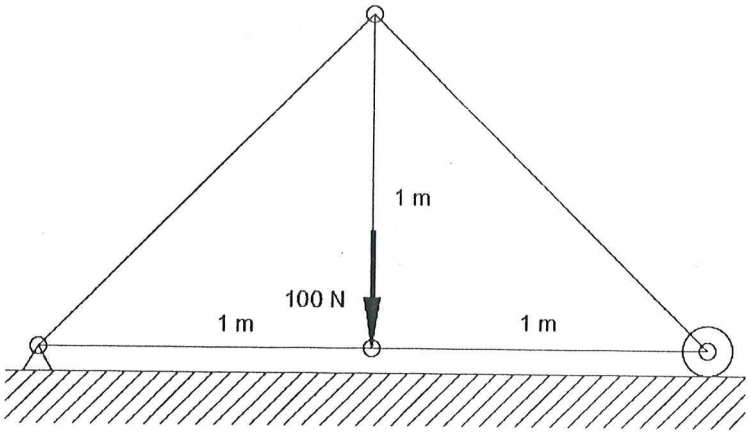

Figure (5)

6. Write short notes through **mathematical derivations** on the following-
   a. Isoperimetric problem: statement through solution by variational formulation.
   b. Minimization of functionals, first variation, Euler-Lagrange equation.
   c. Why Green's strain is a true strain measure but small strain is not.
   d. Show that CST element justifies its title.