Master of Electronics & Telecommunication Engineering Examination, 2018
(2^{nd} Semester)
Coding Theory

Time: Three hours — Full Marks: 100

Answer any ten questions
All questions carry equal marks
Answer all the parts of a question in the same place

1. a) What do you mean by primitive element? (2)

   b) A and B are the two elements of $GF(2^4)$ and 4-tuple representations of $A = (a_3\ a_2\ a_1\ a_0)$ and $B = (1\ 1\ 0\ 0)$. Derive a simplified expression of AB for a given reduction polynomial $r(x) = x^4 + x^3 + 1$ (4)

   c) Simplify following two expressions
   i) $\alpha^7 + \alpha^3 + \alpha$
   ii) $(x + \alpha^3)^5 (x + \alpha^{10})$
   where α is a primitive element over $GF(2^4)$, such that $\alpha^4 + \alpha^3 + 1 = 0$.
   (2+2)

2. a) For an (n, k) block code derive the expression for Hamming bound. (4)

   b) If the generator matrix G of a (7, 4) Hamming code is

$$G = \begin{bmatrix} 1\,0\,0\,0 & 1\,0\,1 \\ 0\,1\,0\,0 & 1\,1\,1 \\ 0\,0\,1\,0 & 1\,1\,0 \\ 0\,0\,0\,1 & 0\,1\,1 \end{bmatrix}$$

   then construct corresponding generator matrix of a (8, 4) Hamming code. Also determine the outcome of the decoder when the received codeword is $v = (0\ 0\ 1\ 0\ 1\ 0\ 1\ 0)$.
   (2+4)

3. a) Determine the parity-check polynomial of the (7, 4) cyclic code with generator polynomial $g(x) = x^3+x+1$. Hence find the generator polynomial, the block length and the information length of the cyclic code that is dual to the (7, 4) code. (3+2+1)

   b) Determine the systematic and non-systematic codeword polynomials for the (7, 4) code generated by $g(x) = x^3 + x + 1$ and information block $i = (1\ 1\ 0\ 0)$. (4)

4. a) What is minimal polynomial? Find the minimal polynomial of α^7, where α is a primitive element over $GF(2^4)$, such that $\alpha^4+\alpha+1 = 0$. (1+4)

   b) If the minimal polynomials of α, α^3 and α^5 are $\phi_1(x) = x^4 + x + 1$, $\phi_3(x) = x^4+x^3+x^2+x+1$, and $\phi_5(x) = x^2+x+1$ respectively, then determine the generator polynomial of a BCH code of $d_{min} \geq 7$. (5)

5. a) The parity check polynomial of a (7, 4) cyclic code is given by $h(x) = x^4+x^2+x+1$. Find the corresponding parity check matrix in systematic form. (5)

   b) What do you mean by syndrome? For an (n, k) systematic cyclic code shows that computation of syndrome polynomial employing g(x) is easier than by h(x), where g(x) and h(x) are generator and parity check polynomials.

(1+4)

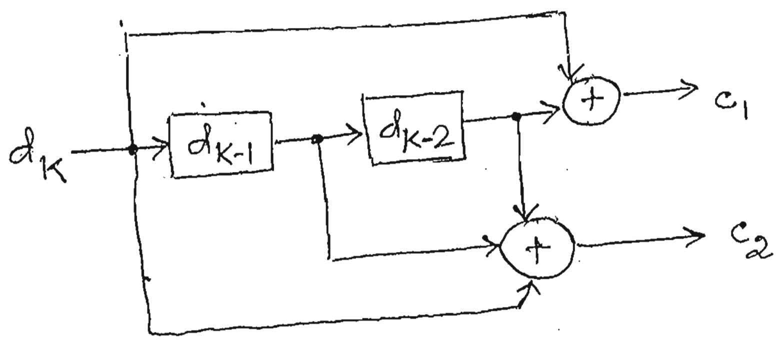
6. If $v(x) = x^9+x^8+x^6+x^4+1$ is a received codeword polynomial of a BCH(15, 7) code over $GF(2^4)$ that has incurred 2 errors, then determine corrected codeword. The field is defined by the primitive polynomial $p(x) = x^4+ x+1$ (10)

7. a) What do you mean by non-binary error correcting codes? Write the properties of Reed-Solomon (RS) code. (1+2)

   b) Construct the generator polynomial of a RS(15, 13) error correcting code and determine the non-systematic codeword corresponding to an information block ($0\ 0\ \alpha\ 0\ 0\ 1\ \alpha^7\ \alpha^2\ 0\ 0\ 1\ \alpha\ \alpha^2$) where α is a primitive element over $GF(2^4)$, such that $\alpha^4+ \alpha^3+1= 0$ (1+4)

   c) How syndromes of RS codes are computed employing Horner's rule (2)

8. a) Explain the operation of the following convolutional encoder employing state diagram. (6)



   b) What types of codes are used for correcting both random as well as burst errors? Explain the basic principle of interleaved codes.

(1+3)

9. a) Maximal length sequence (m-sequence) of period "N" is generated by using Linear Feedback Shift Register (LFSR). Find the expression of Generating Function G(D) of the sequence, where "D" represents the Delay Operator. Which factors does this Generating Function depend on? (6+2)

b) Verify the randomness property of m-sequence related to run length of ones and zeros considering a 15-length m-sequence "111100010011010". (2)

10.a) Consider that "a" is an m-sequence of length 15 given by "010110010001111". Find out the small set of Kasami sequences generated from sequence "a". (7)

b) Comment on the correlation properties of the generated sequences. (3)

11.a) State the importance of generating variable length orthogonal codes for W-CDMA mobile communication system. (2)

b) Develop a code tree structure for the generation of variable length orthogonal codes considering a set of four binary spreading codes each having a chip length of four. Mention the features of the codes generated at different levels. (6+2)

12. Generate the modified version of the existing Walsh Hadamard code set by exploiting the concatenation and permutation properties of codes considering Walsh Hadamard code set as the basic kernel. Note down your observation in this regard. (7+3)