Ref. No.: Ex/PG/ETCE/T/111A/2018

Master of E. & Tel. E. Examination, 2018

(1st Semester)

STATISTICAL COMMUNICATION THEORY

Time: **Three** hours

Full Marks: **100**

Answer Q.1 and any **four** from the rest

Answer must be written at one place for each attempted question

Q1. a. Define the following terms

i) Random process
ii) Independent random processes and orthogonality
iii) Stationary Random process
iv) Ergodic random process
v) Auto correlation and auto covariance of random processes

2x5=10

b) Given the random process X(t)=10 cos(100t+Θ) where Θ is uniformly distributed random variable in the interval (-π, π). Show that the process is correlation ergodic. What will happen to the RP if Θ is a uniform random variable within (0, π/2).

4+2

c) Write the properties of autocorrelation function. 04

Q2. a) A random variable Y is linearly estimated from the observation of another random variable X. Find the mean square estimation error in terms of correlation coefficients. 07

b) A discrete-time random process x(n) is generated as follows:

$$x(n) = \sum_{k=1}^{p} a(k)\, x(n-k) + w(n)$$

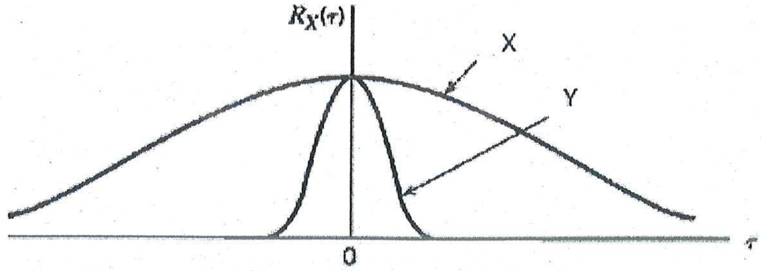
Where w(n) is a white noise process with variance σ_w^2. Another process z(n) is formed by adding noise to x(n),

$$z(n) = x(n) + v(n)$$

Where v(n) is a white noise with variance σ_v^2 and is uncorrelated with w(n).

(i) Find the power spectrum of x(n)
(ii) Find the power spectrum of z(n) 08

2c)



The ACF of two RPs X and Y are drawn in the above figure. Comment on this figure and draw the power spectral density curve for X and Y in the same figure. Explain the curve you have drawn. 05

Q3. a) X(t) is a RP, filtered by linear time invariant filter and generates an output process Y(t). Find the mean of output process, cross correlation between input and output process and the autocorrelation of the output process. If h(n) of the filter is real, then what will be the power spectrum of Y(t) in the z-domain? 10

b) i) Find the power spectrum given the autocorrelation function

$r_x(k) = 2\,\delta(k) + \cos(\pi k/4)$

ii) Find the autocorrelation function given the power spectral densities

$P_x(e^{jw}) = 1/(5 + 3\cos w)$ 2x3=6

c) The input to a linear shift-invariant filter with unit sample response

$h(n) = \delta(n) + 1/2\,\delta(n-1) + ¼\,\delta(n-2)$

is a zero mean WSS process with $r_x(k) = 0.5^{|k|}$, what is the variance of the output process? 04

Q4. a) Define regular process. Show that for any regular process spectral factorization can be performed. From this realization define *innovation representation* and *whitening filtering*. 08

b) Define ARMA (p,q) process. What is the significance of representing such process? 04

d) Consider a first–order AR process that is generated by the difference equation

$Y(n) = a\,y(n-1) + w(n)$

Where, w(n) is a zero mean white noise random process with variance σ_w^2, $|a|<1$. Find the unit sample response of the filter that generates y(n) from w(n) and the autocorrelation and power spectrum of y(n).

08

Q5. a) For a noncausal IIR Wiener filter, design the unit sample response in the mean square error sense. Find also the mean square error for this filter. 10

b) We observe a signal x(n), in a noisy environment as

$$y(n) = x(n) + 0.8\, x(n-1) + v(n)$$

Where v(n) is white noise with variance $\sigma_v^2 = 1$ that is uncorrelated with x(n). The autocorrelation function for WSS process x(n) is $r_x(k)$ $[4,2,1,0.5]^T$. Find the non-causal IIR filter H(z) that produces minimum mean square estimate of x(n). 10

Q.6 a) Discuss the importance of discrete Kalman filtering process. With respect to time and measurement updates derive the different steps of this filtering. Highlight the difference with Wiener filtering process. 10+2

b) Consider the ARMA (1,1) process y(n) given by y(n) + a y(n-1) = v(n) + b v(n-1), where v(n) is the zero mean white noise with variance σ_v^2. Write the state space representation [X(n) , Y(n)]of this process. If the error covariance p(n|n) is given as $P = \sigma_w^2 \begin{bmatrix} b^2 & b \\ b & b^2 \end{bmatrix}$, find the steady state Kalman Gain K. 08

Q7. a) What is called Hypothesis testing? Define binary and multiple Hypothesis testing? 05

b) Let Z_1, Z_2, Z_3......Zn are random variables with zero mean and unity variance. The density function of Z_i under the H_0 follows Gaussian $f_G(z_i)$ and that for H_1 follows Laplacian $f_L(z_i)$.
Design the Hypothesis testing rule according to maximum a posteriori probability criterion (MAP). Design the likelihood ratio test (LRT) according to Bay's rule. 05

c) Define the probability of false alarm and probability of miss detection in any decision criteria. How do they influence to the average cost of any decision? 05

d) When is Neyman-Pearson test applicable? What is the basis of this test? 05

Q8. a) What is called linear prediction? Derive the Wiener-Hopf equation for a first order linear predictor $[W(z) = w(0) + w(1)\, z^{-1}]$ when the measurement of x(n) is noisy such that y(n) =x(n) +v(n), v(n) is white Gaussian noise with variance σ_v^2. 08

b) How is Maximum Likelihood Receiver designed? From this, design the minimax receiver and discuss the effect of the threshold on the design. 12