Ex/PG/ETCE/T/129D/2018

M.E. ELECTRONICS AND TELE-COMMMUNICATION ENGINEERING
FIRST YEAR SECOND SEMESTER-2018

PATTERN RECOGNITION **Time: 3 hours** **Full Marks: 100**

Answer **Q. 1** and **any five** from **the rest.**

1. Answer the following questions: **5 x 3**
   a) Find the variance of a uniform random variable x in $[a, b]$.
   b) Mathematically express the computational structure of a first-order Hidden Markov Model.
   c) Mathematically show how Parzen windows can be used for density estimation.
   d) State Mercer's Theorem.
   e) Distinguish between k-means clustering and Forgy's method of clustering.

2. a) Mathematically represent a d-dimensional normal density with a clear explanation of all the symbols. **2**
   b) Calculate $P(-0.2 \le x \le 0.5)$ where x has the triangular density: **4**

$$p(x) = \begin{cases} 1-x & if\ 0 \le x \le 1 \\ x+1 & if\ -1 \le x \le 0 \\ 0 & otherwise \end{cases}$$

   c) Explain the principle of maximum likelihood estimation of parameters in a statistical distribution. How this is different from method of the parameter estimation using moments? **3+2**
   d) Find the maximum likelihood estimate for μ in a normal distribution. **6**

3. a) Feature x is normally distributed for class A with $\mu = 0$ and $\sigma = 1$. The same feature is also normally distributed for class B with $\mu = 2$ and $\sigma = 2$. What is the probability that the sample with $x = 1$ belongs to class B? **6**
   b) Find the optimum decision regions for the problem in a). **5**
   c) What is least-risk decision boundary? Mathematically show how the least –risk decision boundary can degenerate to the optimal decision boundary in the Bayesian way. **2+4**

4. a) How the Hidden Markov Models (HMM) can help in parametric decision making? Mention any two pattern recognition applications where HMM can be useful. **2+2**

b) State the learning problem in HMM. Provide a detailed solution to this problem using Baum-Welch algorithm. 2+6

c) Obtain the overall probability of classification error using a model based error estimation strategy. 5

5. a) Show that p(x) = k/NV as a general formulation of the non-parametric density estimation problem (symbols have their usual meanings). 5

b) Given dataset $X = \{4,5,5,6,12,14,15,15,16,17\}$, estimate the density of $p(x)$ using Parzen window at

i) $y = 3$ with a window size of 4

ii) $y = 3$ with a window size of 8

iii) $y = 7$ with a window size of 4 2x4

iv) $y = 7$ with a window size of 8

c) Discuss how smooth kernel functions can improve density estimations as compared to that of Parzen windows. 4

6. a) Consider classifying a sample using kNN algorithm with a feature vector (1, 1) given samples (3, 0), (4, 1) and (3, 2) from class A and samples (1, -1), (1, -1.5) from class B. Make separate decisions with values of k as 1 and 3 and Euclidean distance in both the cases. 5

b) Show that the nearest neighbor error rate is bounded by twice Bayes' error rate. 5

c) Explain the importance of kernelized SVM. 3

d) Consider the following mapping: $x \in R^2 \rightarrow y \in R^3$ where

$$y = \begin{bmatrix} x_1^2 \\ \sqrt{2}\, x_1 x_2 \\ x_2^2 \end{bmatrix}$$

Show that $y_i^T y_j = (x_i^T x_j)^2$. What is the importance of this result? 3+1

7. a) Define Fisher's Discriminant Ratio. 2

b) Describe the dimensionality reduction approach of LDA with necessary mathematical details. 7

c) Show how LDA can also be used as a classifier. 2

d) Explain how Karhunen-Loeve transform of a vector can be obtained from its autocorrelation matrix. How can this be of help for dimensionality reduction? 4+2

8. a) Perform a hierarchical clustering of the following data using the complete linkage algorithm and Euclidean distance. Show the distance matrices and the dendogram. 8

Sample	x	y
1	0.0	0.0
2	0.5	0.0
3	0.0	2.0
4	2.0	2.0
5	2.5	8.0
6	6.0	3.0
7	7.0	3.0

b) Perform a partitional clustering of the above data points in a) using the k-means algorithm with k =2. Use the first two samples as the seeds. Show necessary steps. **8**

c) Differentiate between hierarchical and paritional clustering. **1**