Ex:/CON/MATH/T/125/2019(OLD)

BACHELOR OF CONSTRUCTION ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - II E

Time : Three hours Full Marks : 100

Symbols/Notations have their usual meanings.
Answer any ***ten*** questions.

1. (a) Show, by vector method, that the diagonals of a rohmbus intersect at right angles.

   (b) Determine the unit vector, which is perpendicular to the vectors $\hat{i}+2\hat{j}+\hat{k}$ *and* $\hat{i}+\hat{j}-3\hat{k}$.

   (c) Show that $\left(\vec{a}\times\vec{b}\right)^2 = |\vec{a}|^2\,|\vec{b}|^2 - \left(\vec{a}\,.\,\vec{b}\right)^2$. 4+3+3

2. (a) Show that the straight line joining the mid points of two non-paralalel sides of a trapezium is parallel to the parallel sides and half of their sum.

   (b) Show that the necessary and sufficient condition for two proper vectors to be parallel is that their cross product vanishes.

(Turn Over)

(c) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$,

find t such that $\vec{a} = t\vec{b}$ is perpendicular to $\vec{c}$. 4+3+3

3. (a) Find a such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$, and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar.

(b) For any three vectors $\vec{a}, \vec{b}, \vec{c}$ show that

$\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]$ 4+6

4. (a) Find the equation of the plane which passes through the point (3, –3, 1) and is parallel to the plane 2x+3y+5z+6=0.

(b) Find the equation of the plane which cuts off intercepts a, b, c from the axes. 5+5

5. (a) Show that the lines

$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}, \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar.

subject to $5x_1 + 2x_2 \leq 45$

$4x_1 + 5x_2 \leq 53$

$x_1 \geq 2$

$x_1, x_2 \geq 0.$

Find also the minimum value of z.

(b) Show that Var(X) = F(X^2) – $\{E(X)\}^2$. 7+3

12. (a) The probability density function of a random variable x is (fx) = k(x–1) (2–x) for $1 \leq x \leq 2$.

Determine (i) the value of k (ii) $P\left(\frac{5}{4} \geq x \leq \frac{3}{2}\right)$.

(b) The mean and s.d. of a Binomial distribution are respectively 4 and $\sqrt{\frac{8}{3}}$. Find the values of n and p. Hence evaluate P(X= 0) 6+4

8. Obtain the various possible solutions of the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables. Identify the solution which is appropriate with the physical nature of the equation. Justify your answer. 10

9. Solve the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t > 0$, subject to the conditions $u(0,t) = 0$ and $u(L,t) = 0$ for $t > 0$; and $u(x,0) = 3 \sin n\pi x$. 10

10. Solve the two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 \le x \le a,\ 0 \le y \le b,$$

subject to the conditions :

$u(0,y) = 0$, $u(a,y) = 0$ for $0 < y < b$.

$u(x,0) = 0$, $u(x,b) = f(x)$, for $0 < x < a$. 10

11. Solve the following LPP by graphical method :

Maximize $z = -2x_1 + 5x_2$

(b) Find the magnitude and the equations of the shortest distance between the lines

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \text{ and } \frac{x-2}{3} = \frac{y-1}{-5} = \frac{x+2}{2}$$ 4+6

6. (a) Form a PDE by eliminating the arbitrary constants a, b and c from

$$(x-a)^2 + (y-b)^2 + z^2 = c^2$$

(b) Form a PDE by eliminating the arbitrary functions from $z = yf(x) + xg(y)$.

(c) Form a PDE by eliminating the arbitrary function f from $f(x+y+z,\ x^2+y^2+z^2) = 0$ 3+3+4

7. (a) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

(b) Solve : $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. 5+5