Ex./CON/MATH/T/112/2019(OLD)

BACHELOR OF CONSTRUCTION ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester, Old Syllabus)

Mathematics - I E

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Show without expanding

(i) $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

(ii) $$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

(iii) $$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & a+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

5+5+5

(Turn over)

(b) Solve by Crammer's rule

2x+3y+z = 11, x+y+z = 6, 5x – y+10z = 34. 5

2. (a) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}$. 5

(b) If $A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{pmatrix}$ show that $A = A^{-1}$. 4

(c) Find the rank and normal form of the matrix

$$\begin{pmatrix} 2 & 0 & 2 & 2 \\ 3 & -4 & -1 & -9 \\ 1 & 2 & 3 & 7 \\ -3 & 1 & -2 & 0 \end{pmatrix}$$ 7

(d) Show that the system of equations is inconsistent.

x – 2y + z – w = –1

3x – 2z + 3w = – 4

5x – 4y + w = – 3 4

(b) If $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$, $(x,y) \neq (0,0)$

$= 0$, $(x,y) = (0,0)$

then show that $\dfrac{\partial^2 f}{\partial x \partial y} \neq \dfrac{\partial^2 f}{\partial y \partial x}$ at (0,0). 6

(c) Let $\phi(x,y,z) = x^2 + y^2 + xz$. Find the directional derivative of ϕ at the point P(2,–1,3) in the direction of the vector $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$. 6

8. (a) Let $\vec{F}(x,y,z) = 2xz\,\hat{i} - x\hat{j} + y^2\hat{k}$. Evaluate $\iiint_V \vec{F}.dv$ where V is the region bounded by the surfaces x = 0, y = 0, y = 6, $z = x^2$ and z = 4. 10

(b) State Gauss' Divergence Theorem. Use it to evaluate $\iint_S \vec{F}.\overrightarrow{ds}$ where $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. 2+8

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5. (a) State and prove Darbaux's Theorem. 6

(b) Evaluate (if possible)

(i) $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$

(ii) $\int \frac{x\,dx}{x^4+1}$ 5+4

(c) State and prove Lagrange's Mean Value Theorem. 5

6. (a) Let S and T be any two non-empty sets. Show that there is a bijection between $S\times T$ and $T\times S$. 10

(b) If $f(h)=f(0)+hf^{/}(0)+\frac{h^2}{2}f^{//}(\theta h)$, $0<\theta<1$, find the value of θ when h=1 and $f(x) = (1-x)^{5/2}$. 5

(c) If $\lim_{x\to 0}\frac{\sin 2x + a\sin x}{x^3}$ exists then find the value of 'a' and also find the limit. 5

7. (a) State and prove Euler's theorem for homogeneous functions of degree n. 8

3. (a) Solve the following systems of equations :

(i) $x + y + z + w = 0$
$x + 3y + 2z + 4w = 0$
$2x + z - w = 0$

(ii) $x + y + z = 6$
$x + 2y + 3z = 14$
$x + 4y + 7z = 30$

5+5

(b) Find the eigen values and eigen vectors of

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$ 7

(c) Define the Riemann integral $\int_a^b f(x)dx$. 3

4. (a) Without integrating prove that

(i) $\int_0^\pi x\,sin\,x\cos^2 x\,dx=\frac{\pi}{3}$

(ii) $\int_0^1 \frac{\log(1+x)}{1+x^2} = \frac{\pi}{8}\log z$ 5+6

(b) Let f be a bounded function defined on [a,b]. For any two partitions P and Q of [a,b], prove that $L(P,f) \le U(Q,f)$. 6

(c) Give an example of a function which is not Riemann integrable. 3

(Turn over)