Ref. No *Ex/PG/EE/T/127E/2018*

MASTER OF ELECTRICAL ENGG., SECOND SEMESTER EXAMINATION, 2018

OPTIMAL AND ROBUST CONTROL

Time: Three Hours

Full Marks: 100

Answer any **FIVE** questions.

1. a) Find the 2-norm and ∞-norm of the following signal: 8+6+6

$$u(t)=\begin{cases} 0, & if \quad t\le 0 \\ 1/\left(2\sqrt{t}\right), & if \quad 0<t\le 1 \\ 0, & if \quad t>1 \end{cases}$$

b) For the system with transfer function $G(s)=\dfrac{2(1+s)}{(1+0.5s)(1+0.2s)}$, find $\|G\|_\infty$.

c) State the methods used to compute $\|G\|_\infty$ for a system with given transfer function $G(s)$.

2. a) Enumerate the possible sources of parameter perturbation in a control system. 5+5+10

b) State Kharitonov's Theorem for an n^{th} order system.

c) For the unity feedback system of a damped rotating gun turret, the forward path transfer function is given by

$$G(s)=\frac{K}{(1+sT)}\cdot\frac{1}{s(s+a)}$$

The nominal system parameters are a=5; T=0.1; K=2. Investigate the robust stability of the closed-loop system for ±10% variation in each of these parameters.

3. a) For a unity feedback control system, the plant transfer function is given by $P(s)=\dfrac{10}{s(s+2)}$ and the forward path proportional controller transfer function is given by $C(s)=10$. (8+7)+5

Find

(i) the largest value of the complementary sensitivity M_t and the corresponding frequency ω_{mt}.

(ii) the allowable size of the process uncertainty ΔP.

b) Briefly explain how the method of M-circles may be used to calculate the permissible values of gain and phase variations for a system in the presence of uncertainty.

4. a) Prove that the distance from -1 to the Nyquist plot of L equals $1/\|S\|_\infty$, where L is the loop transfer function and S is the sensitivity function for any system. 10+10

b) For the system shown in Fig. 1, with plant $P(s)$, controller $C(s)$, $F(s)=1$ and $r=0$, find the $\|H\|_\infty$ norm when the input is chosen as $w=(-n, d)$ and the output is chosen as $z=(x, v)$.

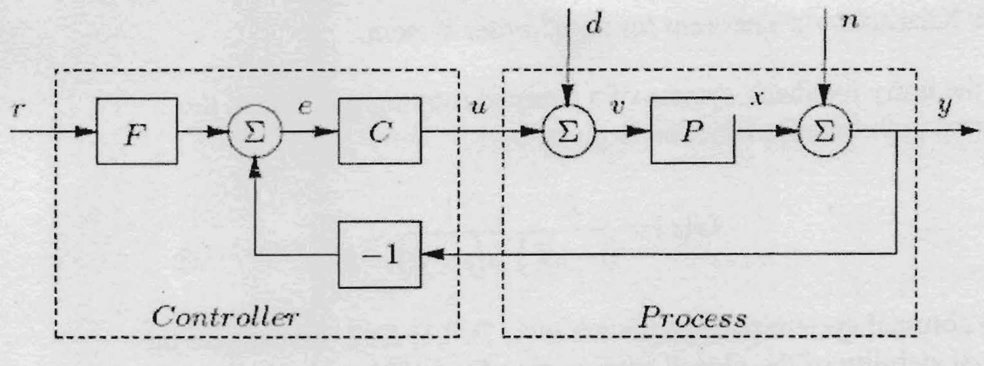

Fig. 1

5\. a) Briefly explain the following: (2X3)+8+6

(i) Brachistochrone problem

(ii) Geodesic problem

(iii) Isoperimetric problem

b) Find a curve $y(x)$ which gives an extremum value to the functional

$$J = \int_0^1 \left(1 + y'^2\right) dx \quad \text{with } y(0) = 1,\ y(1) = 2.$$

c) Derive the condition for the extremum of a functional dependent on higher order derivatives.

6\. a) Derive the expression for the extremum of the functional 8+6+6

$$J = \int_{x_1}^{x_2} F\left(x; y; y'\right) dx \quad \text{for the variable end-point problem.}$$

b) What are transversality conditions?

c) Using transversality conditions, prove that the line lying on the line of centers will be the shortest distance between two circles.

7\. a) With the help of one example in each case, briefly explain *five* different types of optimal control problems. 10+10

b) Using the Hamilton-Jacobi equation, show that an optimal control law for the continuous LQR problem is obtained from the solution to the matrix Riccati equation.

8. a) Explain what you understand by the term *Quadratic performance index* with the help of an example. 5+(8+7)

b) A regulator contains a plant that is described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -2x_1 - 3x_2 + u$$
$$y = x_1$$

and has a performance index given by

$$J = \int_0^\infty \left\{ x_1^2 + 2x_2^2 + u^2 \right\} dt .$$

(i) Find the elements of the Riccati matrix P in the steady state.

(ii) Design an optimum controller for the above system.