

Master of Computer Science & Engg. 1st Semester Examination, 2018

Information and Coding Theory

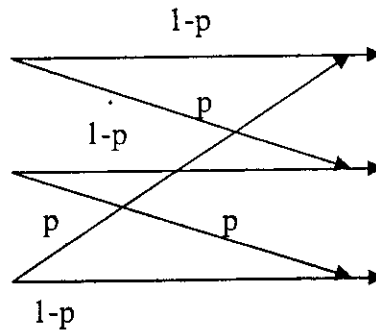
Full marks: 100

Time: 3 hours

Attempt any five questions

1. a) What is information theory? How to measure information associated with an event? Find out the expression for entropy of a source. 7
- b) Consider X and Y are two sources. Show that $H(X) \geq H(X|Y)$. 6
- c) consider the joint probability matrix P(X,Y) as follows.
- | | | | |
|----|------|------|------|
| | y1 | y2 | y3 |
| x1 | 3/40 | 1/40 | 1/40 |
| x2 | 1/20 | 3/20 | 1/20 |
| x3 | 1/8 | 1/8 | 3/8 |
- Find the conditional probability matrix P(Y|X). 4
- d) A source, X produces four symbols with probabilities 1/2, 1/4, 1/8 and 1/8. Find $H(X^3)$. 3
2. a) Suppose X and Y are two random variables with outcomes x_i and y_j respectively (i varies from 1 to n and j varies from 1 to m). Show that $I(x_i; y_j) = I(y_j; x_i)$.
A binary channel with error probability p has two source symbols x_0 and x_1 . Corresponding source symbol probabilities are 3/4 and 1/4. Received symbols are y_0 and y_1 . Find out $I(x_0; y_0)$. 6
- b) Plot transfer rate of a BSC with error probability (assume, source symbols are equi-probable) and also with source symbol probability (assume, constant error probability). 6
- c) Consider a BSC with error probability p. Suppose a block of n bits are transmitted and receiver is capable of correcting m or fewer errors only. What is the probability that receiver provides incorrect output? 3
- d) Show that cascading of BSC increases transmission error. 5
- 3: a) What is prefix code? State Kraft's inequality and prove its sufficiency. 8
- b) Consider prefix code for the symbols of a source X. Show that average code length cannot be smaller than H(X). 4
- c) A source generates six symbols with probabilities 1/2, 1/8, 1/8, 1/16, 1/8 and 1/16. These are to be encoded with Huffman coding (using binary symbols). Find the code for the symbols and coding efficiency. 8

4. a) What is Markov source? Obtain the conditional entropy for such a source. What is the adjoint of a Markov source? 10
 b) Consider a channel as follows:



p is the error probability. Find the channel capacity with necessary justification for your answer. 10

5. a) Describe error control strategies. 4
 b) What is block code? What are the specialties of linear and cyclic code? 4
 c) Explain the outcomes of channel decoder in a communication link? Find out the probability of the decoder outcomes for (5, 4) even parity block code. Assume, the probability for an erroneous bit is 0.01. 8
 d) What is a perfect code? 4

6. a) For a (7, 4) linear code assume the parity equations are as follows.
 $u_0+u_1+u_2$, $u_1+u_2+u_3$ and $u_2+u_3+u_0$ where u_i are the message bits. i) Find the generator matrix in systematic form and also find the parity check matrix, ii) Draw the encoder circuit, iii) Assume the message bits are 1011. What is the codeword? Assume, second received bit from right is erroneous. Find the syndrome. 4+3+4
 b) Codeword of (n, k) linear code is transmitted over BSC with transition probability p . What is the probability that error goes undetected at the receiver? 3
 c) Show that, no two elements in the standard array are same. In standard array based decoding, explain what measures you will take to minimize the decoding error. 6

7. a) Consider $v(x)$ is a code polynomial of (n, k) cyclic code and $v^{(i)}(x)$ is the code polynomial obtained after i number of cyclic shift on $v(x)$ towards right. Find the algebraic relation between them. 3
 b) Show that for (n, k) cyclic code, the degree of generator polynomial is $n-k$. 6
 c) Consider in (n, k) cyclic code, $s(x)$ be the syndrome of received code polynomial $r(x)$. Then find out the syndrome for $r^{(1)}(x)$. 4