M.E. COMPUTER SCIENCE & ENGG., 1st Year 2018
1st Semester
THEORY OF COMPUTING

Time: Three hours | Full Marks: 100

Answer any ***five*** questions

1. (a) Design a single tape Turing machine to accept the language L of all palindromes of *odd* length over $\{a, b\}$ with necessary justifications and flowchart.

   (b) Explain the scheme of choosing the names of states and additional symbol names for this machine.

   (c) Explain carefully how it rejects palindromes of *even* length. Hence prove that it will not accept any string outside L.

   11+4+5

2. (a) Let L be a language which is accepted by a Turing machine M_1 with 1-way infinite tape. Now another Turing machine is M_2 is constructed which has exactly the same transitions as M_1, but a 2-way infinite tape. Explain if M_1 and M_2 will accept the same language.

   (b) Prove that a language is accepted by a Turing Machine with 2-way infinite tape if and only if it is accepted by a Turing Machine with 1-way infinite tape.
   Please illustrate your construction with necessary diagrams.

   5+15

3. (a) Prove that the complement of a *recursive* set is also *recursive*.

   (b) Prove that the union of two recursively enumerable sets is also recursively enumerable.

   (c) Prove that if a language L and its complement $\bar{L}$ are both recursively enumerable, then L is recursive.

   5+5+10

4. (a) Explain whether the set of all Turing machines over a given input alphabet Σ is an *infinite* set.
   Describe a scheme for binary encoding of all Turing machines over a given input alphabet Σ.
   Prove the *uniqueness* of the coding scheme.

   (b) Hence prove that there exists a language which is not recursively enumerable.
   Explain if this language is infinite.

   13+7

5. (a) Explain if the **Post Correspondence Problem** $\begin{bmatrix}10\\101\end{bmatrix}, \begin{bmatrix}011\\11\end{bmatrix}, \begin{bmatrix}101\\011\end{bmatrix}$ has a solution.

   (b) Show the Post Correspondence Problem $\begin{bmatrix}1\\111\end{bmatrix}, \begin{bmatrix}10111\\10\end{bmatrix}, \begin{bmatrix}10\\0\end{bmatrix}$ has a solution but not as a Modified Post Correspondence Problem.

   (c) Prove that if there is an algorithm for solving the Post Correspondence Problem, then there is an algorithm for solving the Modified Post Correspondence Problem as well.

   4+6+10

6. (a) Prove that for each Turing Machine, there exists an equivalent Turing Machine which never writes a blank or moves to the left of its initial position.

   (b) Define vertex cover, maximal matching and maximum matching for a graph with suitable examples.
   Describe an approximate algorithm for the vertex cover problem.
   Find out the approximation factor of this algorithm with necessary proof.

   7+13

~~7. (a) Define mapping reducibility of languages.~~
Let $A \leq_m B$.
Prove that if A is undecidable, then so is B.

(b) Prove that if the Halting Problem of Turing Machines is undecidable, then so is the Post Correspondence Problem.

5+15