MASTER OF COMPUTER SCIENCE AND ENGINEERING EXAMINATION, 2018

(1st Semester)

ADVANCED ALGORITHMS

Time: Three Hours

Full Marks 100

Answer any 5 questions.

1. (a) How do you measure the *work done* by an algorithm? What do you mean by the statement "the running time of an algorithm is $\Theta(n^2)$"? Explain why the statement, "the running time of an algorithm is at least $O(n^2)$" is meaningless.
   (b) Using *potential method*, show that the total amortized cost of performing *n* increment operations on a binary counter is $O(n)$.
   (c) Write an algorithm to insert an element in a dynamic table. Using aggregate analysis find the amortized cost per insert operation in the dynamic table.

   (3+3+2)+5+7=20

2. (a) What do you mean by "average case analysis"? Explain the average behaviour of a binary search algorithm.
   (b) Use recursion tree method to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/2 \rfloor) + n$. Use the substitution method to verify your answer.
   (c) With at least two examples explain when *Master theorem* would not be applicable to solve the recurrences.

   8+8+4=20

3. (a) What is a randomized algorithm? What is the difference between a Monte Carlo algorithm and Las Vegas algorithm? Give an example of each (first explain the problems and then provide the algorithms).
   (b) Write an approximation algorithm for *Travelling Salesperson* problem. Prove that the algorithm is a polynomial time 2-approximation algorithm.

   (2+3+6)+(3+6)=20

4. (a) What is the importance of decision problems? What is the relationship between decision problems and optimisation problems?
   (b) What do you mean by NP-completeness? Why is it important to show that a problem is NP-complete? How do you prove that a problem is NP-complete?
   (b) Prove that *Circuit-satisfiability* is NP-complete.

   4+6+10=20

5. (a) Write an exact algorithm for subset sum problem.
   (b) Write an approximate algorithm for subset sum problem.
   (c) Show how the exact algorithm works for a set of integers {1, 3, 7, 8, 10} and a target 18.

(d) Show how the lists of integers are reduced in case of the approximation algorithm. Consider all the iterations. Take a suitable value of ε for the Trim function.
(e) What do you mean by *duality* in linear programming? Find the dual of the following problem:

maximise x1 + x2, such that
-5x1 + 2x2 <= 2
4x1 – x2 <=8
2x1 + x2 <=10
x1 >= 0, x2 >= 0

3+3+4+4+6=20

6. (a) Draw the KMP flowchart with fail indexes for the pattern "queue".
(b) Using the two heuristics of Boyer-Moore algorithm, show the values in the **charjump**, **sufx**, and **matchjump** arrays for the pattern "queue".
(c) Using a diagram, explain what values exactly the **matchjump** array contains.

6+10+4=20

7. (a) Why do we need an abstract machine model for parallel algorithm analysis?
(b) Explain a PRAM model for parallel computation. Write a sorting algorithm based on PRAM model. What type of PRAM did you use? What are the other types of PRAM models?

(c) A calculator company produces scientific calculators and programmable calculators. Long-term projections indicate an expected demand of at least 100 scientific and 80 programmable calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170 programmable calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators much be shipped each day. Each scientific calculator is sold at Rs. 20 loss, but each programmable calculator generates a Rs. 50 profit.

(i) Formulate a linear programming problem to find how many calculators of each type must be produced to maximise the profit.

(ii) Show the feasible region graphically and find the maximum profit from the graph.

2+8+10=20

8. (a) Prove that CNF-SAT is NP-complete by reducing *circuit-satisfiability* problem to CNF-SAT.
(b) Prove the three cases of Master Theorem for exact powers.

10+10=20