M. C. E. EXAMINATION, 2018

1st semester

THEORY OF ELASTICITY AND ELASTIC STABILITY

Time 3 hours Full marks 100

Answer **4 questions** taking *at least two questions from Q1, Q 2 and Q3* and *at least one question from Q4 and Q5*. Each question carries 25 marks.

1. At a certain point in a solid metal cube, the stress-components relative to axes x_i are σ_{11} = 60 MPa, σ_{22} = 65 MPa, σ_{33} = 20 MPa, σ_{12} = 25 MPa, σ_{23} = 10 MPa, and σ_{31} = 40 MPa.
   (a) Determine the stress vector on a plane normal to the vector $\vec{R} = 3\hat{i} + 3\hat{j} + \hat{k}$.
   (b) Determine the principal stresses and principal direction of the major principal stress. 9+16=25

2. Derive the Jacobian matrix for strain. Explain the significance of the determinant of the Jacobian matrix. Obtain the strain compatibility relations and explain their utility. The strain components of a point are given by $\varepsilon_x = 5xy + 6z$, $\varepsilon_y = 3xy^2 + yz$, $\varepsilon_z = x^2z + y$, $\gamma_{xy} = 2xy^2$, $\gamma_{yz} = yz^2$, and $\gamma_{zx} = xz^2$. Verify whether the compatibility conditions are satisfied at the point (1, -2, 3). 8+2+8+7=25

3. Derive the transformation relationship for stresses between two coordinate systems x_i and x_i'. How can one obtain the normal and shear stresses on a plane with known direction cosines from the family of stresses σ_{ij}. Explain how many independent material constants are necessary to define the stress -strain laws for anisotropic, monoclinic and orthotropic materials? 6+6+13=25

4. With neat diagrams obtain the governing equation for plate buckling. If a simply supported plate is loaded in one direction by in-plane compressive forces comment on its buckling behavior. 17+8 = 25

5. Deduce the central difference expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and $\frac{d^4y}{dx^4}$. Using these formulae, and making use of the Richardson's extrapolations, find the buckling load for a column fixed at one end and hinged at the other with length L and flexural rigidity EI using h=L/3 and h=L/4. 3+3+3+5+6+5=25