A STUDY OF CERTAIN GENERALIZED KINDS OF CHARACTERIZED SUBGROUPS AND THEIR IMPLICATIONS

Abstract

The thesis is concerned with certain kinds of generalized characterized subgroups of the Circle group $\mathbb{T}$ and their applications in Trigonometric Series Theory or Fourier Analysis. Due to its strong relation with the torsion subgroup, topologically torsion subgroup, uniform distribution of sequence mod 1 and trigonometric thin set, the characterized subgroup has deep roots in different branches of Mathematics as Topological Algebra, Number Theory and Harmonic Analysis. However the study of generalized characterized subgroups has recently gained attention of researchers due to its ability to provide more general view in this context. Many problems on generalized characterized subgroups as well as characterized subgroups are still open. In this thesis, some of these open problems of the literature are considered and more general solutions are provided.

The thesis is divided into three parts. Part I of the thesis deals with some generalized notions of convergence where we have discussed many interesting results related to moduler simple density functions and corresponding ideals. These results enable us to construct various generalized characterized subgroups of $\mathbb{T}$ in Part II. In this part we have provided complete description of these generalized characterized subgroups for arithmetic sequences and solved many open problems from literature. In Part III, as an application, we have presented a new class of trigonometric thin sets namely statistical Arbault sets properly containing the class of classical Arbault sets as well as a large subfamily of $\mathbf{N}$-sets. In particular this class happens to properly contain the types of $\mathbf{N}$-sets which have been extensively used in the literature. It is worthwhile to note that this class provides uncountably many $F_{\sigma\delta}$ subgroups which cannot be characterized.

The aim of this thesis is to present an elaborate description of the topic alongside all the new results which would hopefully be useful to not only the experts working in this field, but also a starting point for those who wish to enter this field.

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