Abstract

A Contribution To Complementarity Theory

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Index No. 125/18/Maths./26

Registration Date: 6/6/2018, Registration No: SMATH1112518

The complementarity problem is identified as a mathematical programming problem and provides a framework for several optimization problem. Among many facets of research in complementarity theory, the issue that has received wide attention is the existence of the solutions and development of efficient algorithms for finding solutions. In complementarity theory, many of the available algorithms are developed based on a pivotal kind of technique that converges to a solution with a finite number of steps. The role of the complementary slackness principle is an important consideration in complementarity theory. The complementary slackness principle for more general programming problem is based on the Karush-Kuhn-Tucker condition of optimality. For linear and quadratic programs, the Karush-Kuhn-Tucker optimality conditions finally reduce to the study of linear complementarity problem (LCP) and this observation was the early motivation for studying the linear complementarity problem. More specifically, the problem which can be posed as an LCP includes linear programming, linear fractional programming, convex quadratic programming and the bimatrix game problem. It is well studied in the literature on mathematical programming and a number of applications are reported in operations research, multiple objective programming problem, mathematical economics, geometry and engineering. The linear complementarity problem (LCP) is the problem of finding a complementary pair of nonnegative vectors in a finite dimensional real vector space that satisfies the feasibility condition or to show that no such vector exists.

Many of the matrix classes encountered in the context of linear complementarity problem are commonly found in several applications. Matrix classes characterize properties of the linear complementarity problem and offer certain features from the viewpoint of algorithms. The linear complementarity problem (LCP) is the problem of finding a complementary pair of nonnegative vectors in a finite dimensional real vector space that satisfies the feasibility condition or to show that no such vector exists. The algorithm proposed by Lemke, known as Lemke's algorithm does not solve every instance of the linear complementarity problem and in some instances, the problem may terminate inconclusively without either computing a solution to it or showing that no solution exists. Many of the results of linear complementarity problem can be stated in terms of the value of a matrix game. The principal pivot transform (PPT) is a fundamental concept for

developing many theories and algorithms in complementarity theory and plays an important role in the study of matrix classes.

The idea of nonlinear complementarity problem (NCP) is based on the concept of linear complementarity problem. The fundamental idea of many iterative methods is to solve a problem by tracing a continuous path that leads to a solution of the problem. The homotopy method is proposed for constructive proof of the existence of solutions to system of nonlinear equations, nonlinear optimization problems, Brouwer fixed point problems, nonlinear programming, game problem and complementarity problem.

The results included in this dissertation are divided into eight chapters. Chapter 1 includes the general introduction about the research work along with the required definitions and notations which will be used in the subsequent chapters. Chapter 2 considers the study of hidden Z-matrix in the context of linear complementarity problem. Chapter 3 contains a study of column competent matrix and its matrix theoretic properties. Some new results on w-uniqueness as well as locally w-uniqueness properties in connection with column competent matrices are established. In chapter 4, K-type block matrices are introduced which include two new classes of block matrices namely block triangular K-matrices and hidden block triangular K-matrices. It is shown that the block triangular K-matrices satisfy the least element property and the solution of the linear complementarity problem with K-type block matrices can be obtained by solving a linear programming problem. Chapter 5 deals with the solution approach of the linear complementarity problem as an initial value problem. A new function along with an interior point approach is proposed to trace a path for finding a solution. Chapter 6 contains a solution method for finding the solution of a two-person zero-sum discounted stochastic game with additive rewards and additive transitions (ARAT) structure as an application of LCP. The results in chapter 7 are concerned with the solution approaches to nonlinear complementarity problem using the homotopy approach. A new homotopy function is developed for finding the solution of the nonlinear complementarity problem through a continuous path ensuring the boundedness property of the trajectory obtained from the homotopy function. Chapter 8 contains a formulation of the oligopolistic market equilibrium problem as an application of NCP. In this study, the equivalence between the nonlinear complementarity problem and the system of nonlinear equations is established and a homotopy method with vector parameter is proposed for finding the solution of the oligopoly market equilibrium problem through a system of nonlinear equations.

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28/3/2023

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28/03/2023