ABSTRACT

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TITLE : A method to scan the parameter space of 2-D nonlinear biochemical oscillators and predicting limit cycle boundaries with the help of Renormalization Group.

This thesis is motivated to find a suitable mathematical tool or a method that will facilitate scanning of the entire parameter space of a non-Lienard type non-linear biochemical oscillator. The parameter space of two-dimensional non-Lienard type non-linear oscillators having at most two system parameters guiding the dynamics of the system is divided into two regions by a curve called the stability curve. Values of the system parameters guiding the overall dynamics of the system from one side of this stability curve cause the system to exhibit isolated closed orbits or limit cycles, while parameter values from the other side of the stability curve cause the system to have stable fixed points. On the curve, the system is supposed to undergo a Hopf bifurcation. The developed mathematical tool can help to scan the parameter space and change the parameter values in a suitable desired manner. Sel'kov model of Glycolysis has been taken up to introduce the procedure since the parameter space of this process is suitable for portraying the effectiveness of the method.

We have started out with very general discussions of the system dynamics of two-dimensional non-linear oscillators and how their dynamical characteristics change with the change in parameter values. The various types of fixed points possible in a dynamical system and conditions for the formation of limit cycles have been discussed in the first chapter. The second chapter emphasizes the importance of the Perturbative Renormalization Group. The application of the Perturbative renormalization group leading to the formation of flow equations has been discussed in detail. The advantage of RG in comparison to ordinary perturbation theory has been discussed, along with its usefulness as a probe in proving the applicability of the presently developed method is studied.

The third chapter of this volume throws light on the application of perturbation theory in the case of two-dimensional non-Linear type non-linear oscillators. Perturbative renormalization group has been applied in the case of the Sel'kov model of Glycolysis, leading to amplitude flow equations which give the distance of the center to the point where the limit cycle cuts the X-axis. These results have been predicted for different parameter values inside the limit cycle region of the parameter space. The parameter values have been varied using an external handle in the form of an external parameter `δ' suitably introduced into the system equations so as to be able to scan the parameter space. The method, however, suffers from some restrictive constraints which limit its applicability to the parameter space, as we shall see in detail.

The fourth chapter of the present report proposes the mathematical procedure which eliminates the restrictions on the external parameter `δ', thus bettering our method earlier presented in chapter 3. This procedure is better fitted in the scanning of the parameter space of the two-dimensional non-Lienard type biochemical oscillators to a large extent, as we shall see in detail, and the same has been proven by comparison of results predicted by the Perturbative Renormalization Group and numerical plots. Lastly, the bibliography for the work has been listed.

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