B.E. MECHANICAL ENGINEERING THIRD YEAR SECOND SEMESTER – 2023

Subject: MECHANICAL VIBRATION ANALYSIS **Time: Three Hours** **Full Marks: 100**

The value of the acceleration due to gravity (g) can be taken as 10 m/s², if it is not specified.

Any missing information may be suitably assumed with appropriate justification.

Q1. Answer any one question from this group.

Q1(a). A bullet of mass m = **10 kg** travelling with the velocity v = **50 m/s** strikes and becomes embedded in a massless board supported by a spring of stiffness k = **6.4 × 10⁴ N/m** in parallel with a dashpot with the coefficient of viscous damping c = **400 N·s/m** as shown in **Fig. Q1a**. Determine the time required by the board to reach the maximum displacement and the value of the maximum displacement. **[10]**

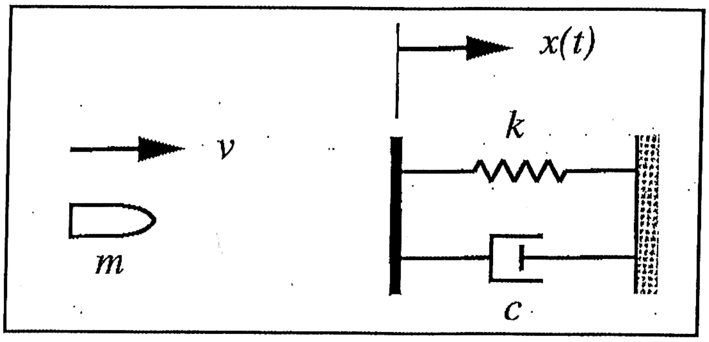


Fig. Q1a

Q1(b). Write the differential equation of motion for the system shown in **Fig. Q1b** and determine the natural frequency of damped oscillation and the critical damping coefficient. Left end of the light horizontal bar is hinged with the wall. **[10]**

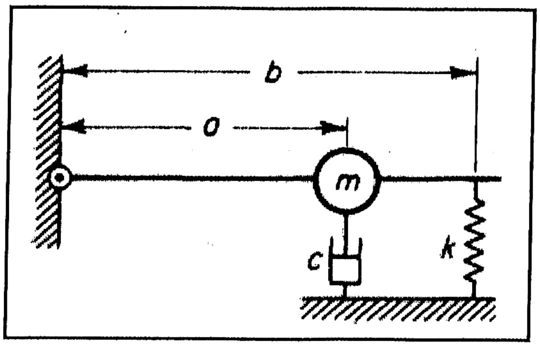


Fig. Q1b

Q2. Answer any two questions from this group.

Q2(a). For a harmonically exited single degree-of-freedom damped spring-mass system, **show that** the maximum value of the frequency response function is observed at a frequency of excitation equal to $\omega = \omega_n\sqrt{1-2\zeta^2}$ and that the maximum value of frequency response function is proportional to $\dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$. **[15]**

Q2(b). A precision grinding machine (as shown in **Fig. Q2b**) is supported on an isolator that has a stiffness of **1 MN/m** and a viscous damping coefficient of **1 kN-s/m**. The floor on which the machine is mounted is subjected to a harmonic disturbance due to the operation of an unbalanced engine in the vicinity resulting into a harmonic motion of the floor as indicated in the figure. Find the maximum acceptable displacement amplitude of the floor if the resulting amplitude of vibration of the grinding wheel is to be restricted to 10^{-6} **m**. Assume that the grinding machine and the wheel are a rigid body of weight **5000 N**. **[15]**

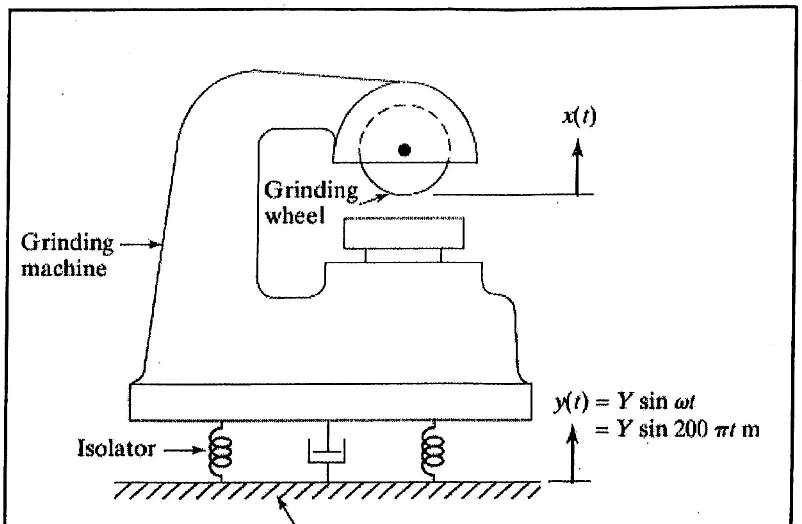
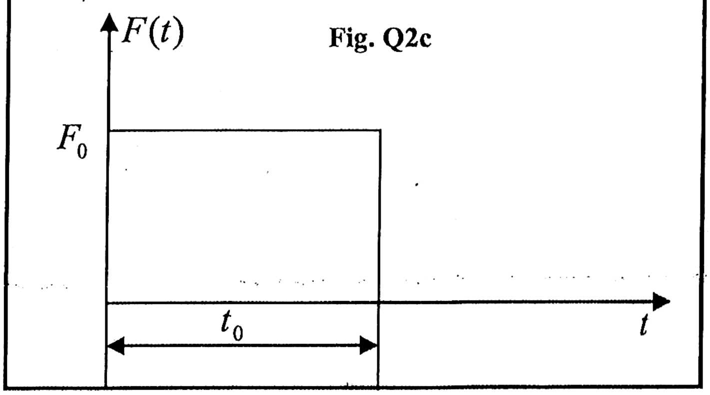


Fig. Q2b

Q2(c). Consider a single-degree-of-freedom undamped vibratory system having a mass m and stiffness k. Using either convolution integral or Laplace Transformation obtain the expression of the transient vibration response of the system when it is acted on by a force as shown in **Fig. Q2c**. Express the result in dimensionless form as $\left(\frac{kx}{F_0}\right)$ for $t < t_0$ and $t > t_0$. **[15]**

Fig. Q2c


Q3. Answer any three questions from this group.

Q3(a). A vehicle as shown in **Fig. Q3a** has a mass of **1000 kg**. The distance between its front wheel axle and the centre of gravity (C.G.) is l_1 = **1** m and that between the rear wheel axle and the C.G. is l_2 = **1.5 m**. The radius of gyration of the vehicle about an axis perpendicular to the plane of the vehicle and passing through its C.G. is **0.9 m**. The equivalent stiffness of the front and rear suspension and wheels are k_f = **18 kN/m** and k_r = **22 kN/m** respectively.

This system can be modeled as a rigid bar of length $l = l_1 + l_2$, supported on two springs k_f and k_r.

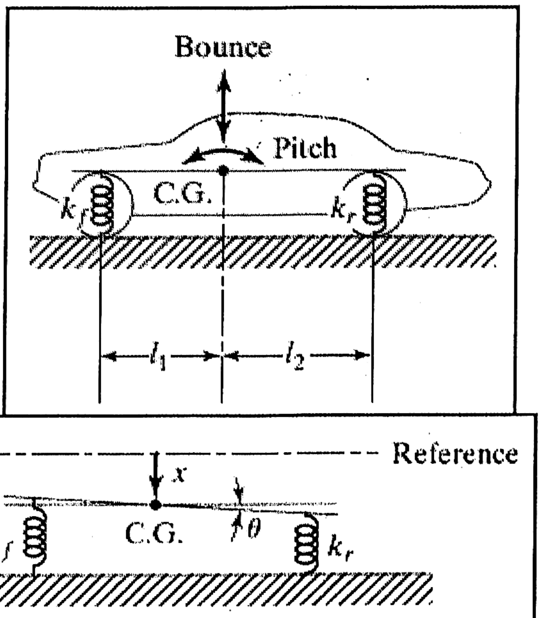


Fig. Q3a

(i) Obtain the differential equations of free vibration of the vehicle for its pitching and bouncing motion in the vertical plane.

(ii) Obtain the corresponding natural frequencies of the system.

(iii) Obtain the ratios of amplitudes of linear and angular motion of the vehicle during the free vibration in normal modes corresponding to the respective natural frequencies. **[15]**

Q3(b). Consider the following equations of motion for free vibration of a two-degree-of-freedom vibratory system:

$[M]\{\ddot{X}(t)\}+[K]\{X(t)\}=\{0\}$ where $[M]=\begin{bmatrix}1 & 0\\ 0 & 4\end{bmatrix}$ and $[K]=\begin{bmatrix}12 & -2\\ -2 & 12\end{bmatrix}$.

(i) Determine the natural frequencies of the system.

(ii) Determine mode shape vectors of the system when they are vibrating in normal modes corresponding to two natural frequencies respectively.

(iii) Determine the expressions of free vibration response of the system with initial conditions: $x_1(0)=x_2(0)=\mathbf{1}$ and $\dot{x}_1(0)=\dot{x}_2(0)=\mathbf{0}$. **[15]**

Q3(c). Consider a multi-degrees of freedom undamped linear vibratory system whose equations of motion during free vibration is given by $[M]\{\ddot{X}(t)\}+[K]\{X(t)\}=\{0\}$, where $\{X(t)\}$ is the $n\times 1$ vector for the system degrees of freedom, $[M]$ is the $n\times n$ mass matrix and $[K]$ is the $n\times n$ stiffness matrix of the system.

Discuss the mathematical procedure to obtain the eigenvalues and eigenvectors of the above equations. Explain how the eigenvalues and eigenvectors of the above equations are related, respectively, to the natural frequencies and mode shapes of the vibratory system.

Also show that the mode shapes corresponding to any two distinct natural frequencies (ω_i and ω_j) of the system are mutually orthogonal to each other. **[15]**

Q3(d). With neat sketches derive the governing differential equation for the free longitudinal vibration of a uniform prismatic elastic bar. When one end of such a bar is fixed and the other end is free, show that the n^{th} normal mode and n^{th} natural frequency are given by respectively,

$$U_n(x)=\sin\left\{\left(\frac{2n+1}{2}\right)\frac{\pi x}{l}\right\} \qquad \text{and} \qquad \omega_n=\left(\frac{2n+1}{2}\right)\frac{\pi}{l}\sqrt{\frac{E}{\rho}}$$

where l is the total length of the bar and x is the coordinate measured along its length, E is the Young's modulus and ρ is the mass per unit volume. **[15]**

Q4. Answer any one question from this group.

Q4(a). Consider that the system shown in **Fig. Q4a** is subject to proportional damping: $c_i = 0.1k_i$, for i = 1, 2. Derive the equations of motion of the system in matrix form. Obtain the frequency response function (FRF) matrix for the system. Obtain the steady-state response of the system due to the harmonic excitation F_2, which is applied on mass m_2, with the following data: m_1 = **9 kg**, m_2 = **1 kg**, k_1 = **24 N/m** and k_2 = **3 N/m**. **[15]**

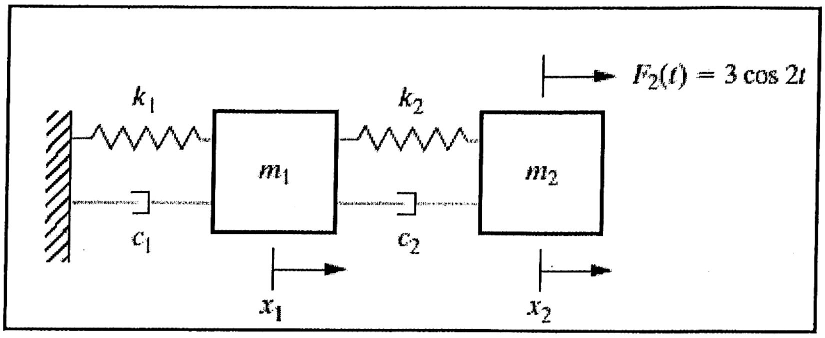


Fig. Q4a

Q4(b). A cart of mass M, which can only oscillate along horizontal direction restricted by two springs of identical stiffness $k/2$ at both sides, carries an inverted pendulum formed by a massless rigid rod, connected to the cart by a d.c. motor and a torsional spring of stiffness k_t, and a lumped mass m at the free upper end of the rod, shown in **Fig. Q4b**. The torsional spring restricts the oscillation of the pendulum about its vertical upright position and the motor applies a control torque τ on the pendulum against its oscillation. If the cart is disturbed by a horizontal force $f(t)$, obtain the equations of motion for small oscillation of the system using **Lagrange's Principle.** Consider x, the displacement of the cart, and θ, **small** angular displacement of the pendulum with respect to the cart, be two generalised coordinates of the system. Comment if the resulting equations of motion are statically/inertially coupled. Express the equations motion in states-space with x, θ, $\dot{x}$ and $\dot{\theta}$ are four state variables. Write the state matrix of the system. If the control torque is determined by proportional-derivative control law (k_p and k_d being proportional gain and derivative gain respectively) applied on θ with reference value of θ to be zero, obtain the modified state-space equation with modified state matrix. **[15]**

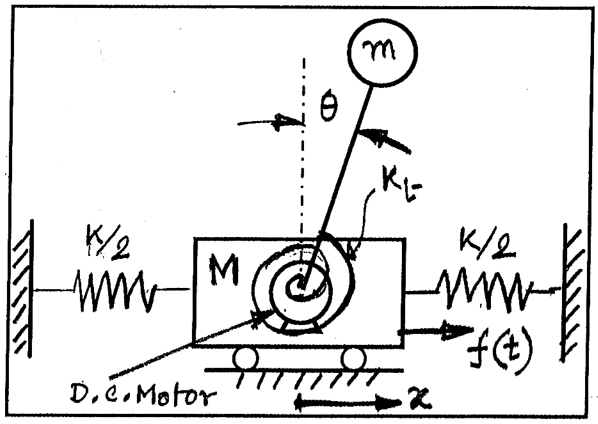


Fig. Q4b