Bachelor of Mechanical Engineering Examination, 2023

(2nd Year, 2nd Semester)

Mathematics-IV

Time : Three hours Full Marks : 100

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (50 Marks)

Answer ***any five*** questions. 10×5=50

All questions carry equal marks.

1. Write short notes on : 2×5

   i) Frequency distribution,

   ii) Histogram and frequency polygon,

   iii) Ogive,

   iv) Independent events,

   v) Sample space

2. a) If A, G and H are respectively the arithmetic mean, geometric mean and harmonic mean of a frequency distribution, show that $A \geq G \geq H$, mentioning the case when equality holds.

   b) Given below is the distribution of 140 candidates obtaining marks X or higher in a certain examination:

X :	10	20	30	40	50	60	70	80	90	100
c.f. :	140	133	118	100	75	45	25	9	2	0

Calculate the mean, median and mode of the distribution. 4+6

3. a) If for a random variable X, the absolute moment of order K exists for ordinary K=1, 2, ..., n, then prove that the following inequalities

i) $\beta_K^2 \le \beta_{K-1}\beta_{K+1}$, ii) $\beta_K^{1/K} \le \beta_{K+1}^{1/(K+1)}$

hold for K=1, 2, 3, ..., n–1, where β_K is the Kth absolute moment about the origin.

b) Prove that for any frequency distribution, standard deviation is not less than mean deviation from mean.

6+4

4. a) Find the mean and central moments of arbitrary order n for the normal distribution with parameter μ and σ. Also find the coeffient of skewness and kurtosis for this distribution.

b) The first four moments of a distribution about the value 4 of the variable are –1·5, 17, –30 and 108. Find the moments about mean and the origin. 4+6

5. a) State the axioms of probability. Show that conditional probability satisfies the axioms of probability.

b) Prove that if A, B and C are random events in a

4. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$f(x) = 0, \quad \text{when } -\pi < x \le 0$$
$$= \frac{\pi x}{4}, \quad \text{when } 0 \le x \le \pi$$

5. a) Find the Fourier series of the function

$$f(t) = 0, \text{ when } -2 < t < -1$$
$$= 3, \text{ when } -1 < t < 1$$
$$= 0, \text{ when } 1 < t < 2.$$

b) Find $L^{-1}\left(\frac{1}{\sqrt{2s+3}}\right)$.

6. Find the Fourier Transformations of the following functions

i) $e^{-|t|}$ ii) $f(t) = 6e^{-8t^2}$

7. i) Find inverse Laplace Transformation of the function

$$F(z) = \frac{z}{z^2 - z + 8}$$

ii) Find the Laplace Transformations of the following functions:

$$f(t) = \frac{3t}{T}, \quad 0 < t < T$$
$$= 1, \quad t > T$$

b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trial? 6+4

Part – II (50 Marks)

Answer ***any five*** questions. 10×5=50

1. a) Find the Z-Transformations of the following functions:

   i) $f(n) = 5n$ ii) $f(n) = 5^n$

   b) Solve the equation using Z-Transformation

   $f(n+1) + 3f(n) = n$, given : $f(0) = 2$.

2. i) Find $L[F''(t)]$, where L stands for Laplace Transformation.

   ii) Solve the equation using Laplace Transformation:

   $y'' + 9y = 0$, given : $y(0) = 0$, $y'(0) = 2$

3. Find the Fourier Transformations of the following function

   i) $f(x) = \text{signx} = 1, \quad \text{when } x > 0,$
   $\quad = -1, \quad \text{when } x < 0.$

   ii) $f(x) = 1, \quad \text{when } |x| \le x_0$
   $\quad = 0, \quad \text{otherwise.}$

sample space and if A, B, C are pairwise independent and A is independent of $(B \cup C)$, then A, B and C are mutually independent. 5+5

6. a) A certain drug manufactured by a company is tested chemically for its toxic nature. Let the event 'the drug is toxic' be denoted by E and the event 'the chemical test reveals that the drug is toxic' be denoted by F. Let $P(E) = \theta$, $P(F/E) = P(\bar{F}/\bar{E}) = 1 - \theta$. Then show that probability that the drug is not toxic given that the chemical test reveals that it is toxic is free from θ.

   b) The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? 5+5

7. a) A player tosses a coin and is to score one point for curvy head and two points for every tail turned up. He is to play on until his score reaches or passes n. If p_n is the chance of attaining exactly n score, show that $p_n = \frac{1}{2}[p_{n-1} + p_{n-2}]$ and hence find the value of p_n.