Ref. No. : Ex/ME(M2)/BS/B/MATH/T/121/2023

B.E. MECHANICAL ENGINEERING EXAMINATION 2023

FIRST YEAR SECOND SEMESTER

Mathematics -II

Full Marks -100 **Time : 3 hr**

Use Separate Answer scripts for each Part.

Part – I (50 Marks)

Answer Question number 1 and any eight from the followings.

1. Find the inverse of $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ 2

2. Reduce the matrix A to row reduced echelon form and hence find its rank
$$A = \begin{pmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{pmatrix}$$
6

3. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 4 & 2 & -1 \\ 7 & 3 & 1 \end{pmatrix}$ by row operations.
6

4. For what values of k, the equations
$x + y + z = 1$,
$2x + y + 4z = k$,
$4x + y + 10z = k^2$,
have solutions?
6

5. If λ be an eigen value of an orthogonal matrix, then $\frac{1}{\lambda}$ is also an eigen value of that matrix.
6

6. Find the eigen values and the corresponding eigen vectors of the matrix
$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$
6

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7. Solve the equations if possible by matrix inversion method
   x + y + z = 8,
   x - y + 2z = 6 ,
   $3x + 5y - 7z = 14,$

6

8. Find the angle between the lines whose direction cosines are given by the relations
   3l + m + 5n = 0 and 6mn - 2nl + 5lm = 0.

6

9. A variable plane which is at a constant distance 3p from the origin cuts the axes at A, B, C. Show that locus of the centroid of the triangle ABC is
   $x^{-}2 + y^{-}2 + z^{-}2 = p^{-}2$

6

10. Find the surface generated by a straight line which intersects the lines $y = 0, z = c$; $x = 0, z = -c$; and the hyperbola $z = 0, xy + c^2 = 0$

6

11. Prove that the centre of sphere which touch the lines $y = mx, z = 0$; $y = -mx, z = -c$; lie on $mxy + cz(1 + m^2) = 0$.

6

12. Find the equation of the plane passing through the line 2x - y = 0 = 3z -y and perpendicular to the plane 4x + 5y -3z = 0 .

6

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B.E. MECHANICAL ENGINEERING EXAMINATION

FIRST YEAR SECOND SEMESTER - 2023

Subject: MATHEMATICS-II

Time: 3 hours **Full Marks: 100**

Part-II (50 Marks)

Use Separate Answer scripts for each Part

Q. No	Answer any *five* questions.	Marks
1.	(a) Verify Stokes' theorem for the vector function $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ round the rectangle bounded by the straight lines $x = 0, x = a, y = 0, y = b$. (b) Find the directional derivative of $\phi = xy^2z + 4x^2z$ at $(-1,1,2)$ in the direction $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.	6+4
2.	(a) If $\vec{F} = 2xz\hat{\imath} + y^2\hat{\jmath} + yz\hat{k}$, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (b) If $\vec{a} + \vec{b} + \vec{c} = 0$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.	7+3
3.	(a) Find the equations of the osculating plane, normal plane, and rectifying plane to the twisted cubic $x = 2t, y = t^2, z = \frac{1}{3}t^3$ at $t = 1$. (b) Show that the vector $\sin y\,\hat{\imath} + \sin x\,\hat{\jmath} + e^z\hat{k}$ is neither solenoidal nor irrotational.	6+4
4.	(a) Find the constants a, b, c so that $\vec{F} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\jmath} + (4x + cy + 3z)\hat{k}$ is irrotational and show that $\vec{F}$ can be expressed as the gradient of a scalar function. (b) If the vectors α and γ are perpendicular to each other, then show that the vectors $\vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$ and $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma}$ are also perpendicular to each other.	7+3

Q. No		Marks
5.	(a) Find the equation of the tangent plane and normal line to the surface $4z = x^2 - y^2$ at $(3,1,2)$. (b) If V and V' be solutions of Laplace's equation such that $\frac{\partial V}{\partial n} = \frac{\partial V'}{\partial n}$ at all points of the closed surface S, then prove that $(V - V')$ is constant at all points inside S.	5+5
6.	(a) Show that if $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$, then $\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 0$. (b) A rigid body is spinning with an angular velocity of 5 radians per second about an axis of direction $(0,3,-1)$ passing through the point $A(1,3,-1)$. Find the velocity of the particle at the point $P(4,-2,1)$.	5+5
7.	(a) If $\vec{F} = (5x^2 + 6y)\hat{\imath} - (3x + 2y^2)\hat{\jmath} + 2xz^2\hat{k}$, then evaluate $\int_C \vec{F} \cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the paths C given by i. $x = t,\ y = t^2,\ z = t^3$ ii. the straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$. (b) Find the unit tangent vector to any point on the curve $x = t^2 - t, y = 4t - 3, z = 2t^2 - 8t.$ ******** —— *******	7+3