Ex/ME/5/MATH/T/111/2023

B.Mechanical(Evening). Examination, 2023
(1ST YR, 1ST SEM)

MATHEMATICS - III

Full Marks : 100 **Time: Three hours**

Part - I

Answer any four questions **$12.5 \times 4 = 50$**

1.(a) Find the Z-Transformations of the following functions:

$$(i)\ f(n) = 9n \qquad (ii)\ f(n) = 4^n$$

(b) Solve the equation using Z-Transformation

$$f(n+1) + 3f(n) = n, \quad given:\ f(0) = 2.$$

2.(i) Find $L[F''(t)]$, where L stands for Laplace Transformation.

(ii) Solve the equation using Laplace Transformation:

$$y'' + 9y = 0\ , \quad given:\ y(0) = 0,\ y'(0) = 2$$

3. Find the Fourier Transformations of the following function

$$(i)\ f(x) = e^{-x^2}$$

$$(ii) f(x) = 1\ when\ \mid x \mid \leq x_0$$
$$= 0,\ otherwisw.$$

[Turn over

4. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$f(x) = 0, \quad when \quad -\pi < x \leq 0$$
$$= \frac{\pi x}{4}, \quad when \quad 0 \leq x \leq \pi$$

5. (a) Find the Fourier series of the function

$$f(t) = 0, \quad when \quad -2 < t < -1$$
$$= k, \quad when \quad -1 < t < 1$$
$$= 0, \quad when \quad 1 < t < 2.$$

(b) Find

$$L^{-1}\left(\frac{1}{\sqrt{2s+3}}\right).$$

Part-II

Answer any two questions: **$15 \times 2 = 30$**

6. (i) Solve the equation:

$$xdy - ydx = \sqrt{y^2 + x^2}dx$$

(ii). Find general solution and singular solution:

$$p = \ln(px - y), \quad where \quad p = \frac{dy}{dx}$$

7. Find the general solution:

$$(D^2 + 2D + 1)y = 4x^2, \quad where \quad D = \frac{d}{dx}$$

8. Define ordinary point and regular singular point of the differential equation

$$P_0(x)y_2 + P_1(x)y_1 + P_2(x)y = 0.$$

Find the series solution near the ordinary point x=0 of the equation

$$y_2 + 3xy_1 + 3y = 0$$

9. Solve the Bessel's differential equation.

$$x^2y_2 + xy_1 + (x^2 - n^2)y = 0.$$

Part-III

Answer the following questions: **10 × 2 = 20**

10. Solve the equations:

(i) $(y+z)p + (z+x)q = x + y$, (ii) $z^2 - pz + qz + (x+y)^2 = 0$

$$\left[where\ \ p = \frac{\partial z}{\partial x}\ ,\ \ q = \frac{\partial z}{\partial y}\right]$$

11. Solve the equation using the method of separation of variables.

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$