Ex/ME/5/MATH/T/121/2023

B.Mechanical(Evening). Examination, 2023

(1ST YR, 2ND SEM)

MATHEMATICS

PAPER - IV

Full Marks : 100 **Time: Three hours**

Part - I

Answer any four questions **$12.5 \times 4 = 50$**

1.(a) Define Median. State it advantages and disadvantages.

(b) From the following distribution of scores calculate the median.

Scores:	6 - 10	11 - 15	16 - 21	21 - 26	26 - 30
Frequency:	8	7	20	6	4

2. Define mode and from the given data, find mode.

Age	No. of persons
20 - 25	50
25 - 30	70
30 - 35	80
35 - 40	180
40 - 45	150
45 - 50	120
50 - 55	70
55 - 60	50

3. What is advantages of standard deviation of a set of observations? From the following distribution , calculate standard deviation :

Class interval :	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency:	2	4	7	4	3

4. (a) Define independent events. Let A and B are two independent events. Show that A^c and B^c are also independent.

(b) If A and B are two events in a sample space S such that

$$P(A) = 0.3,\ P(B^c) = 0.4,\ P(A \cup B) = 0.8.$$

Find

$$(i)\ P(A \cap B),\quad (ii)\ P(A^c \cap B^c),\quad (iii)\ P(A^c \cup B^c)$$

[A^c is complement of A]

5.(a) Define Mean. State it advantages and disadvantages. Also describe it uses.

(b) Find the arithmetic mean from the frequency distribution.

Weight in kg.:	50	55	60	65	70
No. of men:	15	20	25	30	10

6. (a) Two unbiased dice are thrown together at random. Find the expected value of the total number of points shown up.

(b) A random variable has the following probability distribution:

x:	1	2	3
p(x):	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Find the expectation and variance of the random variable x.

7. Let A and B are two events, then prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Using the above result, find

$$P(A \cup B \cup C),$$

for three events A,B and C.

Part - II

Answer any four questions **$12.5 \times 4 = 50$**

1. (a) Define symmetric and skew symmetric matrix. Show that any matrix can be expressed as a sum of a symmetric and a skew symmetric matrix.

(b) Show that product of two orthogonal matrices is orthogonal.

2. Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

Also, find out A^{-1}.

3. Define vector space.
Let

$$S = \{(x, y, z)/x + y + 2z = 0\}.$$

Show that S is a subspace.
Show also that

$$S = \{(x, y, z)/x^2 + y^2 = z^2\}.$$

is not a subspace.

4.

Using Gram Schmidt process, find an orthonormal basis from the set of vectors

$$(1, 0, 1), \quad (1, 0, -1), \quad (1, 3, 4).$$

5. Find an orthogonal matrix which diagonalize the matrix

$$A = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$$

Also, Diagonalise A.

6. What do you mean by subspace of a vector space? State the necessary and sufficient condition for a non empty subset W of a vector space V(F) be a subspace of V. Give an example of subspace. Show that intersection of two subspaces is also a subspace.

7. Define linear mapping and kernel of a linear mapping. Show that $T : R^2 \to R^3$ defined by

$$T(x, y, z) = (x + y + z, 2x + y + 2z, x + 2y + z)$$

is a linear mapping. Find Kernel T and dimension of Kernel T.