Ex/ME/5/Math/T/111/2023(S)

B.Mechanical(Evening) SUPPLEMENTARY EXAM 2023

(1ST YR, 1ST SEM)

MATHEMATICS

PAPER - III

Full Marks : 100 **Time: Three hours**

Part - I

Answer any four questions **$12.5 \times 4 = 50$**

1.(a) Find the Z-Transformations of the following functions:

$$(i)\ f(n) = n \qquad (ii)\ f(n) = a^n$$

(b) Solve the equation using Z-Transformation

$$f(n+1) + 2f(n) = n\ ,\quad given:\ f(0) = 1.$$

2.(i) Find $L[F''(t)]$, where L stands for Laplace Transformation.

(ii) Find the Laplace Transformations of the following functions:

$$\begin{aligned} f(t) &= \frac{t}{T}\ , \quad 0 < t < T \\ &= 1\ , \quad t > T \end{aligned}$$

3. Find the Fourier Transformations of the following functions

$$(i)\ e^{-|t|} \quad (ii)\ f(t) = Ne^{-\alpha t^2}$$

[Turn over

4. State Dirichlet's conditions for convergence of a Fourier series. Find the Fourier series of the function

$$\begin{aligned} f(x) &= 0, \quad when \quad -\pi < x \leq 0 \\ &= \frac{x}{2}, \quad when \quad 0 \leq x \leq \pi \end{aligned}$$

5. (a) Find the Fourier series of the function

$$\begin{aligned} f(t) &= 0, \quad when \quad -2 < t < -1 \\ &= k, \quad when \quad -1 < t < 1 \\ &= 0, \quad when \quad 1 < t < 2. \end{aligned}$$

(b) Find inverse Laplace Transformation of the function

$$F(z) = \frac{z}{z^2 - z + 8}$$

Part-II

Answer any two questions: **15 × 2 = 30**

6. (i) Solve the equation:

$$xdy - ydx = \sqrt{y^2 + x^2}dx$$

(ii). Find general solution and singular solution:

$$p = \ln(px - y), \quad where \ p = \frac{dy}{dx}$$

7. Find the general solution:

$$(D^2 - 2D + 1)y = 5x, \quad where \ D = \frac{d}{dx}$$

8. Define ordinary point and regular singular point of the differential equation

$$P_0(x)y_2 + P_1(x)y_1 + P_2(x)y = 0.$$

Find the series solution near the ordinary point x=0 of the equation

$$y_2 + 3xy_1 + 3y = 0$$

9. Solve the Legendre's differential equation.

Part-III

Answer the following questions: **$10 \times 2 = 20$**

10. Solve the equations:

$$(i)\ z^2 - pz + qz + (x+y)^2 = 0\ , \qquad (ii)\ (ii)\ x^2p + y^2q = z^2$$

$$\left[where\ \ p = \frac{\partial z}{\partial x}\ ,\ \ q = \frac{\partial z}{\partial y}\right]$$

11. Solve the equation using the method of separation of variables.

$$\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial y^2} = 0$$