B.E. MECHANICAL ENGINEERING

THIRD YEAR SECOND SEMESTER SUPPLEMENTARY EXAM– 2023

Subject: MECHANICAL VIBRATION ANALYSIS **Time: Three Hours** **Full Marks: 100**

The value of the acceleration due to gravity (g) can be taken as 10 m/s², if it is not specified.

Any missing information may be suitably assumed with appropriate justification.

Q1. Answer any one question from this group.

Q1(a). A viscously damped system has a stiffness of **5,000 N/m**, critical damping constant of **0.2 Ns/mm**, and a logarithmic decrement of **2.0**. If the system is given an initial velocity of **1 m/s** with zero initial displacement, determine the maximum displacement of the system. [10]

Q1(b). A connecting rod weighing **21.35 N** oscillates **53** times in **1 minute** when suspended as shown in **Fig. Q1b**. Determine its moment of inertia about its centre of gravity, which is located **0.254 m** from the point of support. [10]

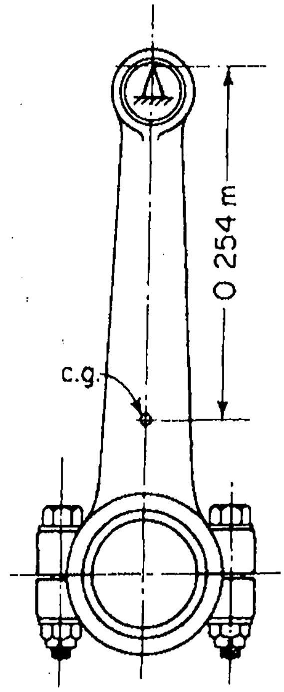

Fig. Q1b

Q2. Answer any two questions from this group.

Q2(a). Find the total response of a single degree-of-freedom damped spring-mass system with $m = 10$ **kg**, $c = 20$ **N·s/m**, $k = 4000$ **N/m**, $x_0 = 0.01$ **m** and $\dot{x}_0 = 0$, when an external force $F(t) = F_0 \cos(\omega t)$ acts on the system with $F_0 = 100$ **N** and $\omega = 10$ **rad/s**. [15]

Q2(b). Consider a spring-mass-damper system which is excited by a harmonic motion ($y = Y \sin \omega t$) at its supporting base as shown in **Fig. Q2b**. Show that the displacement transmissibility of the system is given by

$$T_R = \left|\frac{X}{Y}\right| = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2}\right]^{1/2},$$

where, X is the amplitude of the absolute displacement response of the mass m, ς is the damping ratio and $r = \frac{\omega}{\sqrt{k/m}}$. [15]

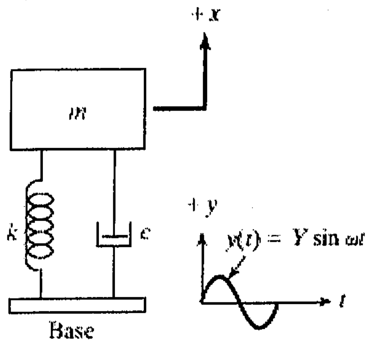

Fig. Q2b

Q2(c). A triangular wave of period T is and is described by $F(t)=\begin{cases}\left(\frac{4}{T}t-1\right) & , \quad 0\le t\le \frac{T}{2}\\ 1-\frac{4}{T}\left(t-\frac{T}{2}\right) & , \quad \frac{T}{2}\le t\le T\end{cases}$

Determine the Fourier coefficients of the wave function. [15]

Q3. Answer any three questions from this group.

Q3(a) Consider the spring-mass system as shown in **Fig. Q3a**. Obtain the equations of motion using Newton's Law. Determine the natural frequencies and the corresponding mode-shape vectors of the system with $m_1 = 2$ **kg**, $m_2 = 8$ **kg**, $k_1 = k_3 = 100$ **N/m and** $k_2 = 20$ **N/m**. Normalize the mode-shapes with respect to mass matrix. [15]

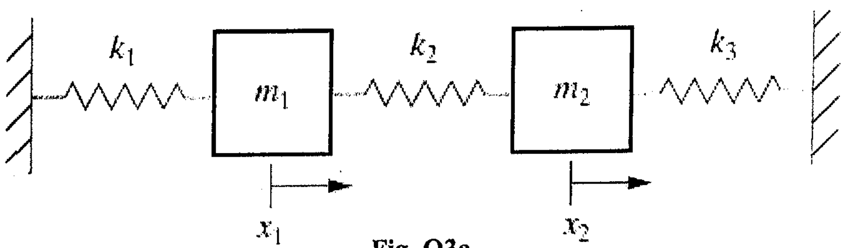

Fig. Q3a

Q3(b). Define the flexibility influence coefficient. Prove Maxwell's Reciprocity Theorem in relation to flexibility influence coefficients. Find the flexibility influence coefficients for the double pendulum as shown in **Fig. Q3b**. Consider the bars connecting masses are massless and the amplitudes of oscillation are small.

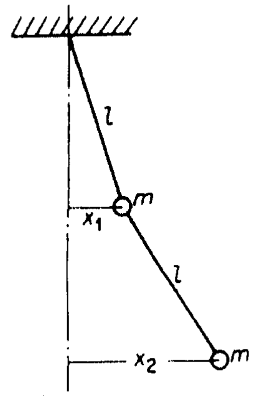

Fig. Q3b

Q3(c). Consider the following equations of motion for free vibration of a two-degree-of-freedom vibratory system:

$[M]\{\ddot{X}(t)\}+[K]\{X(t)\}=\{0\}$ where $[M]=\begin{bmatrix}9 & 0\\ 0 & 1\end{bmatrix}$ and $[K]=\begin{bmatrix}27 & -3\\ -3 & 3\end{bmatrix}$.

(i) Determine the natural frequencies of the system.

(ii) Determine mode shape vectors of the system when they are vibrating in normal modes corresponding to two natural frequencies respectively.

(iii) Determine the expressions of free vibration response of the system with initial conditions: $x_1(0) = \mathbf{1}$ and $x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = \mathbf{0}$. **[15]**

Q3(d). With neat sketches derive the governing differential equation for the free longitudinal vibration of a uniform prismatic elastic beam. When the beam is simply supported, show that the n^{th} normal mode and n^{th} natural frequency are given by respectively,

$$U_n(x) = \sin\left(\frac{n\pi x}{l}\right) \qquad \text{and} \qquad \omega_n = \frac{n^2\pi^2}{l^2}\sqrt{\frac{EI}{\rho}}$$

where l is the total length of the bar and x is the coordinate measured along its length, E is the Young's modulus and ρ is the mass per unit length. **[15]**

Q4. Answer any one question from this group.

Q4(a). A heavy machinery weighing **2.1 × 10⁴ N** has a static deflection of **3 cm**, when it is mounted on a flexible support. The system of the machinery and the support exhibits resonance with a force of amplitude **440 N**. Determine the secondary mass of an undamped vibration absorber which can be designed to limit the maximum deflection of the machinery within **2.5 mm**. **[15]**

Q4(b). Explain with neat sketches the principle of operation of an undamped vibration absorber. Deduce the necessary equations of motion in this process. **[15]**