Ref. No.: Ex/ME(M2)/BS/B/MATH/T/111/2023(S)

B.E. MECHANICAL ENGINEERING
SUPPLEMENTARY EXAMINATIONS - 2023
FIRST YEAR FIRST SEMESTER
Mathematics-I

Time : Three hours **Full Marks:100**

(Notations and symbols have their usual meanings.)

GROUP- A

Answer any five questions from the following.

1. Test wheather the following series converges or not

   (i) $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots$

   (ii) $\sqrt[3]{n^3+1} - n.$

   (iii) $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \ldots$

   3 + 4 +3

2. (i)Show that the sequence $\{x_n\}$, where $x_n = \frac{1}{1.2.} + \frac{1}{2.3.} + \frac{1}{3.4.} + \ldots + \frac{1}{n(n+1)}$ is a bounded monotonic increasing sequence.

   (ii) Examine the convergence of following sequences $\{x_n\}$, where

   (a)$x_n = \frac{(3n+1)(n-2)}{n(n+3)}$

   (b)$x_n = \sqrt{n}$

   5 + 3 +2

3. (i) Suppose a function $f(x, y)$ defined by $f(x, y) = \frac{x^4+y^4}{x-y}, x \neq y$ and $f(x, y) = 0, x = y$. Is $f(x, y)$ continuous at $(0, 0)$?

   (ii) State and prove Lagrange's Mean value theorem.

   5 + 5

4. (i) Using Mean Value Theorem prove that $\frac{2x}{\pi} < sinx < x$, for $0 < x < \frac{\pi}{2}$.

   (ii) Using Lagrange's method of undetermind multiplier find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$.

   5 + 5

[Turn over

5. (i) (b) Expand the function f(x)= sin x in infinite series in powers of x if possible.

(ii) If a function $f(x,y)$ is defined by $f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$, when $x^2+y^2 \neq 0$ and $f(x,y) = 0$, when $x^2+y^2 = 0$, show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

5 + 5

6. (i) State Euler's theorem of homogeneous function of two variables. Using this theorem prove that
$x^2\frac{\delta^2 u}{\delta x^2} + 2xy\frac{\delta^2 u}{\delta x\delta y} + y^2\frac{\delta^2 u}{\delta y^2} = \frac{tanu}{144}(13 + tan^2 u)$, if $u = cosec^{-1}[\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}]^{\frac{1}{2}}$.

(b) Evaluate the limit $\lim_{x\to 0}(sinx)^{(2tanx)}$

6 + 4

7. (i) If v be a function of r alone, where $r^2 = x^2 + y^2 + z^2$. Show that $\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} + \frac{\delta^2 v}{\delta z^2} = \frac{\delta^2 v}{\delta r^2} + \frac{2}{r}\frac{\delta v}{\delta r}$.

(ii) If $u = log(x^3+y^3+z^3-3xyz)$, then show that $\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} + \frac{\delta u}{\delta z} = \frac{3}{x+y+z}$.

5 + 5

GROUP- B

Answer Question Number 8 and any *four* questions from the rest.

8. Define Riemann Integration of a bounded function $f(x)$ in [a,b].

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9. (a) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of *Beta* function and hence evaluate $\int_0^1 x^5(1-x^3)^{10}dx$.

(b) Evaluate $\int_0^\infty 4x^4 e^{-x^4} dx$.

7 + 5

10. (a) Give an example of a bounded function which is not R-integrable.

(b) Suppose $f(x) = x$ and $g(x) = e^x$,verify the first Mean Value Theorem of Integral Calculus for the interval [-1,1].

5 + 7

11. Examine the convergence of following integrals (any two)

(a) $\int_1^\infty \frac{dx}{x^{\frac{1}{3}}(1+x^{\frac{1}{2}})}$

(b) $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

(c) $\int_a^\infty e^{-x}\frac{sinx}{x^2}dx, a > 0$

6 + 6

12. (a) Evaluate $\iint xy(x+y)dxdy$ over the area bounded by $y = x^2$ and $y = x$.

    (b) Evaluate $\int_0^{\pi} \int_0^{a(1+cos\theta)} r^3 sin\theta cos\theta d\theta dr$.

6 + 6

13. (a) Determine the length of one arc of the cycloid $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$.

    (b) Find the area of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$.

6 + 6

14. (a) Find the surface of the solid generated by revolution of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis.

    (b) Show that $\Gamma(1/2) = \sqrt{\pi}$.

7+5