B.E. INFORMATION TECHNOLOGY SECOND YEAR, FIRST SEMESTER EXAM 2023

Time: Three hours **MATHEMATICS FOR IT- I** Full Marks-100

CO1: Explain and illustrate sum and product of vectors with related applications

Attempt any two (2) questions. 2x5 = 10

I. Show that three points with position vectors **a,b,c**are collinear if and only if there exist three non-zero scalars α,β,γ, $\alpha \neq \pm\beta$, such that αa + βb + γc= 0 and $\alpha + \beta + \gamma = 0$.

II. Let P and Q be diametrically opposite points and R any other point on a sphere. Show that P R and QR are at right angles.

III. Find the volume of a tetrahedron.

CO2: Solve homogeneous, non-homogeneous linear ordinary differential equations of the 1st order and higher orders having constant and variable coefficients and system of lineardifferential equations

Attempt any three (3) questions 3x5=15

I. Solve $xdx + ydy + \frac{xdy-ydx}{x^2+y^2} = 0$ given that y=1 when x=1.

II. Solve $x^2(p^2 - y^2) + y^2 = x^4 + 2xyp$ where $p = \frac{dy}{dx}$.

III. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + cosx$.

IV. Solve by the method of variation of parameters, $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

V. Solve the following system of equations.

$$\frac{d^2x}{dt^2} - 2\frac{dy}{dt} - x = e^t \cos t$$
$$\frac{d^2y}{dt^2} + 2\frac{dx}{dt} - y = e^t \sin t$$

CO3: Express a given real-world problem as a linear programming problem and use simplexmethod to solve it

Attempt any eight (8) 8x5 =40

I. A company has two grades of inspectors, I and II, who are to be assigned for a quality control inspection. It is required that at least 2000 pieces be inspected per 8-hour day. Grade I inspectors can check pieces at the rate of 50 per hour with an accuracy of 97%. Grade II inspectors can check pieces at the rate of 40 per hour with an accuracy 95%. The wage rate of Grade I inspectors is Rs. 4.50 per hour and that of Grade II is Rs. 2.50 per hour. Each time an error is made by an inspector, the cost to the company is one rupee. The company has available, for the inspection job, 10 Grade I and 5 Grade II inspectors. Formulate the problem to minimize the total inspection cost.

II. Solve the following two problems graphically and comment on the optimal values.

Minimize $Z = 3x_1 + x_2$
Subject to
$2x_1 + 3x_2 \geq 2$
$x_1 + x_2 \geq 1$
$x_1, x_2 \geq 0$

Maximize $Z = 2x_1 + x_2$
Subject to
$2x_1 + x_2 \leq 3$
$3x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$

III. If a set of vectors is linearly independent, then prove that any non-empty subset of the given set is always linearly independent.

IV. Find the basic solutions of the following set of equations and classify the solutions as feasible, degenerate and non-degenerate.

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

[Turn over

V. Write the initial table to solve the following problem using Simplex method.

Minimize $Z = 3x_1 - 4x_2 - x_3$

Subject to $x_1 + 3x_2 - 4x_3 \leq 12$

$2x_1 - x_2 + x_3 \leq 12$

$x_1 - 4x_2 - 5x_3 \geq 5$

$x_1 \geq 0$, x_2 and x_3 are unrestricted in sign.

VI. If any variable of the primal problem is unrestricted in sign, then prove that corresponding dual constraint is an equation.

VII. Find the dual of the following LPP.

Maximize $Z = 6x_1 + 5x_2 + 10x_3$

Subject to $4x_1 + 5x_2 + 7x_3 \leq 5$

$3x_1 \quad + 7x_3 \leq 10$

$2x_1 + x_2 + 6x_3 = 20$

$2x_2 + 9x_3 \geq 5$

$x_1, x_3 \geq 0$, and x_2 is unrestricted in sign.

VIII. Suppose an LPP is solving by Big-M method due to the existence of the artificial variable. At the optimal stage for the artificial variable(s) different cases may arise. Point out different situations and corresponding conclusion about the solution of the given problem.

IX. The given table represents the second table in the solution of an LPP (maximization) by Big-M method. State the original problem.

		C	3	-1	-1	0	0	-M	-M
Basis	c_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_4	0	10	3	-2	0	1	0	0	-1
a_6	-M	1	0	1	0	0	-1	1	-2
a_3	-1	1	-2	0	1	0	0	0	-1
z_j-c_j		-M-1	-1	-M+1	0	0	M	0	3M-1

X. State and prove the weak duality theorem.

XI. If primal has unbounded solution, prove that dual has no feasible solution.

CO4: Solve transportation problem using suitable methods and test for optimality

Attempt any two (2) 2x10 =20

I. Describe the Matrix minima rule to find the initial basic feasible solution of the transportation problem. Prove that this rule always allocates value into m+n-1 cells. Assume the usual meaning of the symbols.

II. A steel company has three open hearth furnaces and five rolling mills. Transportation costs (rupees per quintal) for transporting steel from the furnaces to the rolling mills are shown in following table.

	M_1	M_2	M_3	M_4	M_5	
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	25
F_3	6	5	4	7	7	15
	4	9	15	8	8	

It is given that

i. from the second furnace at least 5 quintals steel to transported to the second mill.
ii. from the second furnace at least 8 quintals steel to transported to the third mill.
iii. from the third furnace at least 1 quintal steel to transported to the third mill.

Find the optimal transport schedule with transportation cost. [Symbols have usual meaning]

III. If C be the cost matrix of a transportation and a new cost matrix C_1 is computed from C such that C_1=C-p (p is subtracted from each element of C). Find the relation between the optimal solutions obtained using C and C_1.Find an optimal solution and corresponding cost of transportation problem, when initial solution is obtained using VAM.

	D_1	D_2	D_3	D_4	
O_1	1	2	-2	3	70
O_2	2	4	0	1	38
O_3	1	2	-2	5	32
	40	28	30	42	

CO5: Solve assignment problem using suitable methods and examine for optimality

Attempt Q. no I and any one from {II, III} 5+10 = 15

I. Formulate the assignment problem as LPP.

II. Consider the problem of assigning six operators to six machines. The assignment costs in rupees are in the following table. Operator B cannot be assigned to machine 2, operator E cannot be assigned to machine 4 and operator D is assigned to machine 6.

	1	2	3	4	5	6
A	8	4	2	6	1	4
B	0	--	5	5	4	3
C	3	8	9	2	6	4
D	3	4	6	2	1	9
E	4	3	8	--	3	5
F	9	5	1	0	5	3

III. One car is available at each of the stations 1, 2, 3, 4, 5, 6 and one car is required at each of the stations 7, 8, 9, 10, 11, 12. The distances between the various stations are given in the following matrix. How should the cars be despatched so as to minimize the total mileage coverage?

	7	8	9	10	11	12
1	41	72	39	52	25	51
2	22	29	49	65	81	50
3	27	39	60	51	32	32
4	45	50	48	52	37	43
5	29	40	39	26	30	33
6	82	40	40	60	51	30