Ref. No.: Ex/IT/BS/B/T/212/2023(S)

B.E. INFORMATION TECHNOLOGY SECOND YEAR, FIRST SEMESTER SUPPLEMENTARY EXAM 2023

Time: Three hours | **MATHEMATICS FOR IT- I** | Full Marks-100

CO1: Explain and illustrate sum and product of vectors with related applications

Attempt any **three (3)** questions. 3×5 = 15

I. If **a**, **b** be two given non-collinear vectors, then every vector **r**, co-planar with **a** and **b** can be expressed as $x\mathbf{a} + y\mathbf{b} = \mathbf{r}$, where x and y some scalars.

II. Examine whether the vectors 5a + 6b +7c, 7a – 8b + 9c and 3a +20b + 5c (a, b, c being non-coplanar vectors) are linearly independent or dependent.

III. Show that medians of a triangle are concurrent.

IV. Prove that for any triangle ABC, $\mathbf{c^2 = b^2 + a^2 - 2ab\cos C}$ in the usual notation of plane Trigonometry.

V. For two vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} = b_2\mathbf{j} + b_3\mathbf{k}$, prove that $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

CO2: Solve homogeneous, non-homogeneous linear ordinary differential equations of the 1st order and higher orders having constant and variable coefficients and system of lineardifferential equations

Attempt any three (3) questions 3×5=15

I. Find the differential equation of the family of curves $y = e^{mx}$, where m is an arbitrary constant.

II. Solve $\frac{dy}{dx} = \frac{\sin x + x\cos x}{y(2\log y + 1)} = 0$ given that y=1 when x = 0.

III. Solve $\{y\left(1 + \frac{1}{x}\right) + \cos y\}dx + (x + \log x - x\sin y)dy = 0$

IV. Solve $(D^6-1)\,y = 0$ where D having standard meaning.

V. Solve $(x^2D^2 + 2xD)\,y = \log x$, where D having standard meaning.

CO3: Express a given real-world problem as a linear programming problem and use simplexmethod to solve it

Attempt any five (5) questions 5×8 = 40

I. A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats, and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table. Formulate the linear programming model for the problem.

Food type	Yield per unit			Cost per unit (Rs.)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

II. Solve the following problem.

Maximize $Z = 3x_1 + 4x_2$

Subject to

$x_1 + x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$

[Turn over

III. Complete the following table and compute next table, then comment on the solution of the given problem.

		c	0	2	1	0	0	Min ratio
Basis	c_B	b	a_1	a_2	a_3	a_4	a_5	
a_4	0	4	4	0	-4	1	-1	
a_2	2	3	-3	1	2	0	1	
$z_j - c_j$								

IV. Let the following table is an intermediate table of a problem, which is solved by Big-M method. The current problem is the dual of a primal problem. Complete the given and then compute the next table and find the solution and value of the objective function of the of the primal problem.

		c	-24	-18	-5	0	0	-M	Min ratio
Basis	c_B	b	a_1	a_2	a_3	a_4 ($-e_1$)	a_5 ($-e_2$)	a_6	
a_6	-M	2	0	3/2	-3/4	-1	3/4	1	
a_1	-24	1	1	1/2	1/4	0	-1/4	0	
$z_j - c_j$									

V. If any constraint of the primal problem is equation, then prove that corresponding dual variable is unrestricted in sign.

VI. State the assumptions and limitations of the linear programming problem.

VII. Show that the following system of linear equations has degenerate basic feasible solutions only.

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

VIII. State and prove the weak duality theorem.

CO4: Solve transportation problem using suitable methods and test for optimality

Attempt any **two (2)** questions $2 \times 8 = 16$

I. A steel company has three open hearth furnaces and five rolling mills. Transportation costs (rupees per quintal) for transporting steel from the furnaces to the rolling mills are shown in following table. Formulate the problem mathematically.

	M_1	M_2	M_3	M_4	M_5	
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	25
F_3	6	5	4	7	7	15
	4	9	15	8	8	

II. A company has four warehouses and six stores. The warehouses altogether have supply 22 units of a given commodity. Six stores altogether need 22 units of commodities. The costing of shipping one unit of commodity from warehouse i to store j in rupees is given in the following table. Find the initial basic feasible of the problem using VAM method.

		Stores					
		1	*2*	*3*	*4*	*5*	*6*
Warehouses	*1*	*9*	*12*	*9*	*6*	*9*	*10*
	2	*7*	*3*	*7*	*7*	*5*	*5*
	3	*6*	*5*	*9*	*11*	*3*	*11*
	4	*6*	*8*	*11*	*2*	*2*	*10*

III. Consider the transportation problem whose the cost matrix is given below and the initial basic feasible solution is $\{x_{11}=5, x_{13}=15, x_{14}=20, x_{22}=30, x_{23}=0, x_{31}=15, x_{35}=5, x_{41}=10\}$. Find the minimum transportation cost.

	W_1	W_2	W_3	W_4	W_5	Available
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Required	30	30	15	20	5	100 (Total)

CO5: Solve assignment problem using suitable methods and examine for optimality

Attempt any **two (2)** questions $2 \times 7 = 14$

I. A machine tool company decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor and is given in the following table in hundreds of rupees. Formulate the mathematical model for this problem. .

		Contractors			
		1	2	3	4
Subassemblies	1	15	13	14	17
	2	11	12	15	13
	3	13	12	10	11
	4	15	17	14	16

II. Solve the following assignment problem. Also, comment on the existence of the alternative optimal solution.

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

III. Four new machines M_1, M_2, M_3 and M4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of the limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The assignment of machine i to place j in rupees is given below. Find the optimal assignment.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3