

**BETCE EXAMINATION, 2023**  
**(2nd Year 1st Semester)**  
**Signals and Systems**

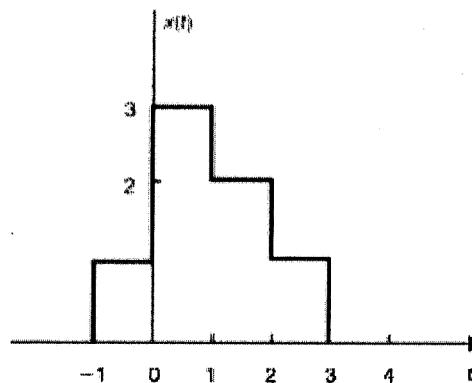
Full Marks: 100

Time: 3 hours

*Answer all parts & subparts of a question under a unit serially in the same place*

**UNIT-1 (Marks: 25)**

- Q1.** a) Sketch the signal  $x(t) = r(t) - r(t-1) - u(t-1) + r(t-2) - r(t-3) - u(t-3)$  where  $r(t)$  and  $u(t)$  represents unit ramp and unit step function respectively. [5]
- b) Check whether the given signal  $x(t) = t u(t)$  is energy or power signal. [5]
- c) Express the given signal  $x(t)$  in terms of unit step functions. [5]



**OR**

- Q1.** a) find whether the given signal  $x(t)$  is periodic or not. If periodic, find the time period of the signal.

$$x(t) = \cos\left[\left(\frac{1}{3}\right)t\right] + \sin\left[\left(\frac{1}{4}\right)t\right] \quad [5]$$

- b) Find even and odd components of the signal  $x(t) = \cos t + \sin t + \cos t \sin t$  [5]

- c) Differentiate between Orthogonal and Orthonormal signal set. State the condition under which an Orthogonal set is said to be complete. [4+1]

- Q2.**a) Prove that time domain convolution between two signals leads to frequency domain multiplication. [6]

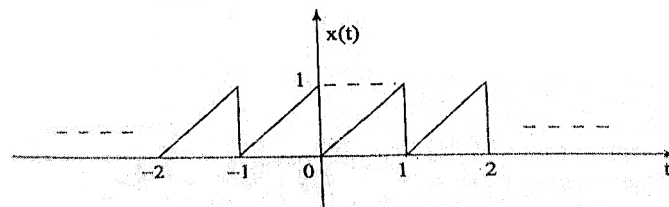
- b) Discuss in brief, the significance of the following terms: [2+2]

[ Turn over

- i) Essential Bandwidth of a signal
- ii) Correlation coefficient between two signals

**UNIT-2 (Marks: 15)**

**Q3.** Find the Exponential Fourier Series for the periodic signal  $x(t)$ . Hence plot the magnitude and phase spectrum of  $x(t)$ . [9+6]



**OR**

**Q3. a)** Find the Fourier Transform of the following signal: [5]

$$x(t) = (1/2) [\delta(t + 1) + \delta(t + 0.5) + \delta(t - 0.5) + \delta(t - 1)]$$

**b)** Explain the significance of Time Shifting property of Fourier Transform. [4]

**c)** The Exponential Fourier Series representation of a signal  $x(t)$  over the interval  $(0, T)$  is

$$x(t) = \sum_{n=-\infty}^{\infty} \left[ \frac{3}{4} + (n\pi)^2 \right] e^{jn\pi t} \quad [3+3]$$

- i) What is the numerical value of  $T$ ?
- ii) One of the components of  $x(t)$  is  $\text{Acos}3\pi t$ . Determine the value of  $A$ .

**UNIT-3 (Marks: 10)**

**Q4.** It is required to reconstruct a practical signal (time limited) from its samples. Explain the effect of Aliasing and the impact of Anti-aliasing filter on the reconstructed signal through neat sketches and necessary labeling of the following (no description is required): [10]

- i) Spectrum of a practical signal
- ii) Spectrum of the sampled signal
- iii) Reconstructed signal spectrum
- iv) Sampling scheme using Anti-aliasing filter

v) Sampled signal spectrum and the reconstructed signal spectrum after incorporation of Anti-aliasing filter

**OR**

**Q4. a)** Draw the waveform of Pulse Position Modulated (PPM) signal considering a sinusoidal message signal and a periodic pulse train as the carrier signal. State one advantage of this kind of modulation. [3+2]

b) The signal  $x(t) = \cos 5\pi t + \cos 10\pi t$  is instantaneously sampled. The interval between the samples is  $T_s$ . Find the maximum allowable value of  $T_s$ . To reconstruct the signal, the sampled signal is passed through a low-pass filter. Find the minimum filter bandwidth to reconstruct the signal without distortion. [5]

**UNIT-4 (Marks: 15)**

**Q5. a)** For the discrete-time system  $y(n) = 2x(n) + 1/[x(n-1)]$ , check whether it is linear/non-linear. Here,  $x(n)$  and  $y(n)$  represents input and output respectively. [5]

b) Compute convolution of two sequences  $x(n)$  and  $h(n)$  graphically. Given,  $x(n) = \{\underline{1}, 2, 4\}$  and  $h(n) = \{\underline{1}, 1, 1, 1, 1\}$ . Underline signifies  $n=0$  position. [10]

**UNIT-5 (Marks: 10)**

**Q6.** If  $f(x)$  is the PDF of a random variable  $X$  defined by

$$f(x) = 2(1-x)^2 \quad 0 < x < 1$$

$$= 0 \quad \text{elsewhere}$$

Find i)  $E[6X + 3X^2]$  and ii) variance of  $2X+3$ . [5+5]

**OR**

**Q6.** The PDF of a continuous random variable  $X$  in the range  $(-3, 3)$  is defined as follows:

$$= (1/16) [3+x]^2 \quad \text{for } -3 \leq x \leq -1$$

$$f_X(x) = (1/16) [2-6x]^2 \quad \text{for } -1 \leq x \leq 1$$

$$= (1/16) [3-x]^2 \quad \text{for } 1 \leq x \leq 3$$

Verify that the area under the curve is unity. Also prove that the mean is zero. [7+3]

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**UNIT-6 (Marks: 25)**

**Q7.** Consider that noise is a random process and it can be represented as the superposition of noise spectral components. Also consider that the spectral component associated with the  $k$ th frequency interval is given by: [15]

$$n_k(t) = a_k \cos(2\pi k \Delta f t) + b_k \sin(2\pi k \Delta f t).$$

Derive the statistical properties of  $a_k$  and  $b_k$ , where the symbols have their usual meanings.

**OR**

**Q7. a)** Derive the expressions for the quadrature components of noise. [8]

**b)** State the properties of the quadrature components of noise. [2]

**c)** Discuss the significance of quadrature component representation of noise. [5]

**Q8.** Consider a cascade of two amplifier stages where  $A_1$  is the gain and  $R_1$  is the total input noise resistance of the first stage;  $A_2$  is the gain and  $R_2$  is the total input noise resistance of the second stage and  $R_3$  is the output resistance. Calculate the equivalent resistance of the cascaded amplifier. Using this knowledge, how can you calculate the noise voltage produced at the output of this cascade? [7+3]

**OR**

**Q8.a)** An RC low-pass filter is placed before the demodulator of a communication system to limit noise in the system. Calculate the amount of noise power which is delivered to the demodulator. [7]

**b)** For low noise level, Noise Temperature is preferred to Noise Figure for the purpose of noise measurement-explain. (No derivation is required). [3]