Ref. No.: Ex/ET/PC/B/T/216/2023

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING

SECOND YEAR FIRST SEMESTER EXAM 2023

Subject: MATHEMATICS III

Time: Three Hours **Full Marks: 100**

Answer Question No. 1 and any FIVE questions from the rest. Q.1 carries 10 marks and the rest are of 18 marks each. Answer in brief with proper justification. All the symbols used carry the standard meanings.

Q. No.		Marks
1.	***Solve with proper reasons.*** (a) Three different numbers are selected at random from the set $A = \{1,2,3,\ldots,11,12\}$. Find the probability that the product of two of the numbers is equal to the third. (b) Evaluate $\lim_{z\to -1+i} \frac{z^2+2z+2}{z^2+2i}$ (c) Determine the constant a so that the vector: $V = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+az)\hat{k}$ is solenoidal. (d) Let X and Y be two random variables such that $Y = a = bX$ where a & b are constants. Find $Var(Y)$. (e) A random variable X has the following pdf: $f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & elsewhere \end{cases}$. Find the value of the constant k.	(2×5) $= 10$
2.	(a) Find the Laplacian of the following scalar field: $i)\ S = x^2y + xyz,\ ii)\ S = r^2z(\cos\phi + \sin\phi)$ (b) Determine the Divergence and Curl of the following vector field: $\vec{\phi} = yz\hat{a}_x + 4xy\hat{a}_y + y\hat{a}_z$ (c) Show that $\boldsymbol{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative field. Find the scalar potential. Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).	(6×3) $= 18$
3.	(a) Using the complex variable technique prove that $\int_0^\infty \frac{\sin mx}{x}dx = \frac{\pi}{2}$ (b) Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^2}$ using Cauchy's integral. (c) Expand $\sin^{-1} z$ in powers of z.	(6×3) $= 18$
4.	(a) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. If $u = 3x + 2xy$, then find v and express $f(z)$ in terms of z. (b) Find the Taylor series expansion of a function of complex variable $f(z) = \frac{1}{(z-1)(z-3)}$ about the point $z = 4$. Find its region of convergence. (c) Find the poles of $f(z) = \frac{(z^2-2z)}{(z+1)^2(z^2+4)}$ and residues at the poles which lie on imaginary axis.	(6×3) $= 18$

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5.	(a) Establish Poisson's Equation and hence Laplace's equation from the fundamental concept of Divergence and the basic equations of Electrostatics. (b) Considering a vector mathematically show how can Divergence Theorem and Stokes' theorem form the basis of conversion of volume integral to surface integral and then to line integral and vice versa. (c) The marks obtained by 1000 students in a final examination are found to be approximately normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be between 60 and 75 both inclusive. Given that area under the normal curve $\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$ between $z = 0$ and $z = 2$ is 0.4772 and between $z = 0$ *and* $z = 1$ is 0.3413.	(6 × 3) = 18
6.	(a) Find the Taylor series expansion of a function of complex variable $f(z) = \frac{1}{(z-1)(z-3)}$ about the point $z = 4$. Find its region of convergence. (b) Find the poles of $f(z) = \frac{(z^2-2z)}{(z+1)^2(z^2+4)}$ and residues at the poles which lie on imaginary axis. (c) i) Evaluate $\lim_{z\to -1+i} \frac{z^2+2z+2}{z^2+2i}$. ii) Prove that $\lim_{z\to 0} \frac{Re(z)}{\lvert z\rvert}$ does not exist.	(6 × 3) = 18
7.	(a) The life in hours of a certain type of four electronic components of a computer follows a continuous distribution given by the density function: $f(x) = \begin{cases} \frac{k}{x^2}, & x \geq 100 \\ 0, & x < 100 \end{cases}$. Find k and determine the probability that all four such components in a computer will have to be replaced in the first 250 hours of its operation. (b) The lifetime of a certain brand of an electric bulb may be considered as a random variable with mean 1200h and standard deviation 240h. Determine the probability using the central limit theorem that the average lifetime of 60 bulbs exceeds 1250h. Given: area under the standard normal curve between $z = 0$ and $z = 1.61$ is 0.4463. (c) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively.	(6 × 3) = 18
8.	(a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given that $u(0,t) = u(l,t) = 0, u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$ where $0 < x < l$. (b) If the random variable X represents the sum of the numbers obtained when 2 fair dice are thrown, determine by Tchebycheff's inequality an upper bound for $P(\lvert X-7\rvert \geq 3)$ and compare it with the exact probability. (c) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$	(6 × 3) = 18