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BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)
FOURTH YEAR
SECOND SEMESTER EXAMINATION 2023

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours **(50 marks for each part)**

Use a separate Answer-Script for each part

PART- I

Answer any *THREE* questions

Two marks reserved for neatness and well organized answers.

1. (a) What is meant by the relative frequency of occurrence of a random event? With help of a suitable example, explain what is meant by 'Statistical Regularity'. How can this phenomenon be utilized to define 'Probability of a random event? 9

   (b) The probability mass function of a random variable X is 7

$$W(x)=\frac{2x+1}{25}; \qquad x=0,1,2,3,4$$

Determine the following probabilities.

(i) $P(X=3)$ (ii) $P(X\le 1)$ (iii) $P(2\le X<4)$

(iv) $P(X\ge -5)$

Also find the expectation of X.

2. (a) State the properties of the cumulative distribution function of random variables. 7

   (b) The cumulative distribution function of a continuous random variable X is 9

$$F(x)=\begin{cases}0 & x<0\\ 0.2x & 0\le x<4\\ 0.04x+0.64 & 4\le x<9\\ 1 & 9\le x\end{cases}$$

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Determine the following.

Probability of $X \leq 6$

(ii) Probability of $X > 5$

(iii) Probability of $-2 \leq X < 8$

(iv) Expectation and mean-square value of *X*.

3. (a) It is claimed that the probability density function of a random variable *Y* is 8

$$f(y) = Ky^2 \text{ for } -1 \leq y \leq 1$$

Find

(i) the value of *K*.

(ii) the probability that ($Y<0.5$).

(iii) The probability that ($-0.25<Y<0.35$)

(b) What is 'Moment Generating Function' of a random variable? How can it be used to obtain the moments of different orders of the random variable? 8

4. (a) Point out the important features related to the shape, size and position of a Gaussian probability density function. 6

(b) The compressive strength of samples of cement canbe modeled by a normal distribution with a mean of 6000 kilogramsper square centimeter and a standard deviation of 100kilograms per square centimeter. 10

(i) What is the probability that a sample's strength is less than 6250 kg/ cm^2.

(ii) What is the probability that a sample's strength is between 5800 and 5900 kg/cm^2 ?

(iii) What strength is exceeded by 90% of the samples ?

Use the attached table for standard Gaussian random variable. Perform linear interpolation if necessary.

5. Write short notes on <u>any two</u> of the following.

(a) **Uniform distribution.**

(b) **Exponential distribution.**

(c) **Markov's inequality and Chebyshev's inequality.**

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F(z)= Probability of $(Z \leq z)$

Table Standard normal distribution **F(z) for z= 0.00– 2.99**

Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

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BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING) FOURTH YEAR SECOND SEMESTER - 2023

SUBJECT: INTRODUCTION TO STATISTICAL & PROBABILISTIC METHODS

Time: Three Hours

Full Marks: 100
(50 Marks for each part)

Use a separate Answer-Script for each part
Two marks for neat and well-organized answers

Question No.	Part-II	Marks

Answer any three questions

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	1.6	1.65	1.70	1.80	1.90	1.95	2.0	2.5
P (z):	0.9452	0.9505	0.9554	0.9641	0.9713	0.9744	0.9772	0.9938
z:	2.55	2.60	2.65	2.70				
P (z):	0.9948	0.9953	0.9960	0.9965				

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1. (a) Establish the relations between the expectation and the variance of the sample mean and the corresponding quantities of the population. Also discuss the significance of the central limit theorem. 8

   (b) An astronomer wants to measure the distance from her observatory to a distant star. However, due to atmospheric disturbances, any measurement will not yield the exact distance *d*. As a result, the astronomer has decided to make a series of measurements and then use their average value as an estimate of the actual distance. If the astronomer believes that the values of the successive measurements are independent random variables with a mean of *d* light years and a standard deviation of 2 light years, how many measurements need she make to be at least 95 percent certain that her estimate is accurate to within ±.5 light years? 8

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2. (a) Explain the significance of the term '95% upper confidence interval' in connection with estimation of the mean of a normal population with known variance. 8

(b) A signal having value μ is transmitted from location A and the value received at B is normally distributed with mean μ and variance 6.25. If the successive received values are 8

6.5, 8.0, 14.5, 9.5, 10.5, 5.0, 7.5, 12.5, 8.5

then determine the 95% confidence interval for μ. Also determine its 99% upper confidence interval.

3. (a) Define the terms, (i) Simple Hypothesis, (ii) Composite Hypothesis, (iii) Type I error, (iv) Type II error, (v) Critical Region. 10

(b) Suppose it is known that if a signal of value μ is sent from station A, then the value received at station B is normally distributed with mean μ and variance 4. It is suspected that the signal value $\mu=8$ will be sent today from A. Test this hypothesis if the same signal value is independently sent five times and the average value received at B is $\overline{X}=9.5$. Consider 5% significance level. 6

4. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. 8

(b) Determine the expected values of Regression Parameters of a simple linear regression model. 8

5. (a) What is "continuity correction"? Explain it with a suitable example. 6

(b) If $\overline{X}$ is the sample mean and S^2 is the sample variance from a normal population with mean μ and standard deviation σ then show that $(n-1)S^2/\sigma^2$ has a chi-square distribution with (n-1) degrees of freedom and $\sqrt{n}(\underline{X} - \mu)/S$ has a t- distribution with (n-1) degrees of freedom where n is the sample size. 10