Ref No: Ex/EE/5/T/421/2023 (S)

BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)
FOURTH YEAR
SECOND SEMESTER SUPPLEMENTARY EXAMINATION 2023

INTRODUCTION TO STATISTICALAND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours **(50 marks for each part)**

Use a separate Answer-Script for each part

PART-I

Answer any *THREE* questions

Two marks reserved for neatness and well organized answers.

1. (a) **Introduce the concept of expectation and variance of discrete random variables. How can they be obtained for continuous random variables ?** 7

(b) **The Department of Electrical Engineering has a lab with six computers reserved for Master's degree students. Let *X* denote the number of these computers that are in use at a particular time of day. Suppose that the probability distribution (i.e. the probability mass function (PMF) of *X* is as given below.** 9

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

where $p(x) = P(X=x)$.
Obtain the probability of at least 3 computers are in use and also the probability that not more than 4 computers are in use. What is the expectation or statistical mean of *X* ?

2. (a) **Suppose the reaction temperature *X* (in °C) in a certain chemical process is uniformly distributed over -5°C to +10°C. Compute** 12

(i) $P(X<0)$
(ii) $P(X>4.5)$
(iii) $P(-3.2<X<6.2)$
(iv) $E[X]$
(v) σ_x

Derive the expression used for (v)

(b) **Check whether or not the following are valid probability density functions. Give explanations.**

[Turn over

(i) $f(x) = -1.5x^2$ **for** $-1<x<+1$ **(ii)** $f(x) = 600x^{-2}$ **for** $100<x<1$ **4**

$= 0$ **otherwise** $= 0$ **otherwise**

3. (a) **What is 'Moment Generating Function' of a random variable? How can it be used to obtain the moments of different orders of the random variable?** **6**

(b) **Derive the expressions for the moment generating function of exponential random variables. Hence obtain the expressions for the expectation and the variance of exponential random variables.** **6+4**

Prove the "Lack-of-Memory" property of exponential random variables.

4. **The life of a type of electricity meter is a normal distributed random variable. The power distribution company of a city installed 2000 such new meters having an average life of 10000 working hours with a standard deviation of 1000 working hrs.** **16**

(i) **How many meters might be expected to fail in 1st 3500 working hours?**

(ii) **In the above problem, how many meters are apprehended to fail between the first 1000 and 8000 working hours ?**

(iii) **After what period of working hours, would we expect 20% meters to have failed?**

Use the attached table. Use linear interpolation wherever necessary

5. **Write short notes on <u>any two</u> of the following.** **8+8**

(a) **Relative frequency of occurrence, statistical regularity and probability of random variables.**

(b) **Properties of cumulative distribution function and probability density function.**

(c) **Markov's inequality and Chebyshev's inequality.**

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F(z)= Probability of $(Z \leq z)$

Table Standard normal distribution **F(z) for z= 0.00– 2.99**

Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

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BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING) FOURTH YEAR SECOND SEMESTER SUPPLEMENTARY EXAM - 2023

SUBJECT: INTRODUCTION TO STATISTICAL & PROBABILISTIC METHODS

Time: Three Hours | Full Marks: 100 (50 Marks for each part)

Use a separate Answer-Script for each part

Two marks reserved for neat and well-organized answers

Question No.	Part-II	Marks

Answer any three questions

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	1.6	1.65	1.70	1.80	1.90	1.95	2.0	2.5
P (z):	0.9452	0.9505	0.9554	0.9641	0.9713	0.9744	0.9772	0.9938
z:	2.55	2.60	2.65	2.70				
P (z):	0.9948	0.9953	0.9960	0.9965				

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1. (a) Show that sample mean is same as population mean whereas its variance is 1/n times the population variance. — 8

   (b) An insurance company has 25,000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million. — 8

2. (a) Explain with the help of a suitable example, what is 'two-sided percentage confidence interval estimate' of the mean μ of a normal population with known variance σ^2. — 8

   (b) A signal having value μ is transmitted from location A and the value received at B is normally distributed with mean μ and variance 6.25. If the successive received values are — 8

   6.5, 8.0, 14.5, 9.5, 10.5, 5.0, 7.5, 12.5 & 8.5,

   then determine the 95% confidence interval for μ. Also determine its 99% upper confidence interval.

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3. (a) What is "continuity correction"? Explain it with a suitable example. 6

(b) The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college. 6

(c) Derive the expression of variance of n number of samples. 4

4. (a) Derive the expression of sample size requirement to meet certain specifications concerning type-II error. 10

(b) Suppose it is known that if a signal of value μ is sent from station A, then the value received at station B is normally distributed with mean μ and variance 4. It is suspected that the signal value $\mu=8$ will be sent today from A. Test this hypothesis if the same signal value is independently sent five times and the average value received at B is $\overline{X}$ =9.5. Consider 5% significance level. 6

5. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. 8

(b) Determine the expected values of Regression Parameters of a simple linear regression model. 8