Ex/EE/5/Math/T/113/2023

Bachelor of Engineering (Electrical) Examination, 2023

(1st Year, 1st Semester)

MATHEMATICS IIF

Time: Three hours Full Marks: 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*

All questions carry equal marks

1(a) Find the matrix A, if $A^2 = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$

(b) Find the inverse of the matrix

$$B = \begin{bmatrix} 1 & 0 & -4 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

(c) Find the rank of the matrix

$$C = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$$

4+8+8

2(a) For what values of x, the following matrix E is singular?

$$E = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

(b) Solve the following system of linear equations by matrix inversion method or Cramer's rule:

$$x + 2y - 3z = 1$$
$$2x - y + z = 4$$
$$x + 3y = 5$$

(c) Find the eigen values of the matrix

$$P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & -3 \end{bmatrix}$$

Also find an eigen vector of P corresponding to its smallest eigen value.

6+6+8

3. Solve the differential equations:

(a) $1+e^{x/y}dx+e^{x/y}(1-x/y)dy=0$

(b) $(x^2+y^2+1)dx-2xy\,dy=0$

(c) $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0$

(d) $(y^2-2xy)dx=(x^2-2xy)dy$

5+5+5+5

4. Solve the following differential equations:

(a) $(D^2-3D+2)y=6e^{-3x}+\sin 2x$

(b) $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=x^2\sin(\log x)$

(c) Solve the following differential equation by the method of variation of parameters:

$$(D^2-6D+9)y=\frac{e^{3x}}{x^2}$$

6+6+8

5(a) (i)Show that $(\vec{a}\times\vec{b})^2=a^2b^2-(\vec{a}.\vec{b})^2$

(ii) If $|\vec{\alpha}|=10, |\vec{\beta}|=1, \vec{\alpha}.\vec{\beta}=6$ then evaluate $|\vec{\alpha}\times\vec{\beta}|$.

(b) Find the constant m such that the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+m\hat{j}+5\hat{k}$ are coplanar.

(c) If $\vec{f}=x^2y\hat{i}-2xz\hat{j}+2yz\hat{k}$ then find curlcurl $\vec{f}$.

8+6+6

6.(a) If $\vec{a}$ is a constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then show that

(i) div $(\vec{a}\times\vec{r})=0$ and

(ii) curl $(\vec{a}\times\vec{r})=2\vec{a}$.

(b) Find the value of the constant λ so that the vector field defined by $\vec{f}=(2x^2y^2+z^2)\hat{i}+(3xy^3-x^2z)\hat{j}+(\lambda xy^2z+xy)\hat{k}$ is solenoidal.

(c) Prove that $div(\phi\vec{A})=grad(\phi).\vec{A}+\phi\,div(\vec{A})$

8+6+6

7. (a) State Caley Hamilton theorem. Use this theorem to compute A^{-1} where

$$A=\begin{pmatrix}2&1\\3&5\end{pmatrix}$$

(b) Solve the equation

$$\begin{vmatrix}x+2&2x+3&3x+4\\2x+3&3x+4&4x+5\\3x+5&5x+8&10x+17\end{vmatrix}=0$$

(c) Find the direction cosines of a line which is perpendicular to the lines whose direction ratios are (-1, 1, 2) and (1,2,3).