Bachelor of Engineering (Electrical Engineering) Supplementary Examination, 2023

(1st Year, 1st Semester)

MATHEMATICS IIF

Time: Three hours Full Marks: 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*

1(a) Express the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 2 \\ 7 & 2 & 5 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

(b) Find the inverse of the matrix

$$B = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(c) Find the rank of the matrix

$$C = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & 4 & 6 & 2 \\ -1 & 5 & 4 & 6 \end{bmatrix}$$

6+7+7

2(a) Find the adjoint of the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & -3 & 1 \\ -1 & -1 & 3 \end{bmatrix}$$

(b) Solve the following system of linear equations by Cramer's rule:

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

(c) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

and hence find A^{-1}.

6+7+7

3. Solve the following differential equations:

(a) $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(b) $(1+xy)ydx + (1-xy)x\,dy = 0$

(c) $(2xy\cos x^2 - 2xy + 1)dx + (\sin x^2 - x^2)dy = 0$

(d) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

5+5+5+5

4. Solve the following differential equations:

(a) $(D^2 - 4)y = e^x + \sin 2x$

(b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$

(c) Solve the following differential equation by the method of variation of parameters:

$(D^2 - 2D)y = e^x \sin x$

6+7+7

5(a) If $\vec{a} = 5t^2\hat{i} + t\,\hat{j} - t^2\hat{k}$ and $\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j}$, find $\frac{d}{dt}(\vec{a}.\vec{b})$ and $\frac{d}{dt}(\vec{a}\times\vec{b})$.

(b) Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

(c) Find the work done in moving a particle in the force field $\vec{f} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from (0,0,0) to (2,1,3).

8+6+6

6.(a) Find the directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

(b) Show that the vector $\vec{f} = (-x^2 + yz)\hat{i} + (4y - z^2x)\hat{j} + (2xz - 4z)\hat{k}$ is solenoidal.

(c) Prove that $curl(\phi\vec{A}) = grad(\phi)\times\vec{A} + \phi\,curl(\vec{A})$

6+7+7

7. (a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

(b) If

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$ then find the values of x.

(c) Find the direction cosines of a line which is perpendicular to the lines whose direction ratios are (-2, 1, -1) and (-3, -4, 1).

8+6+6