B.E. ELECTRICAL ENGINEERING, FOURTH YEAR
SECOND SEMESTER EXAM 2023

ADVANCED CONTROL THEORY

Time: 3 Hours
Full Marks: 100

Answer both parts on the same answer script

Part-I

In this part
Answer Question no. 1 which is compulsory
AND
Answer any one question from Question nos. 2 and 3
AND
Any one question from Question nos. 4 and 5

1. a) Define the term "static nonlinearity". Give two examples of static nonlinearity. **CO1** **[4]**

   b) Sketch the characteristics of a relay with hysteresis and explain why it is called a nonlinearity with memory. **CO2** **[2]**

   c) Define the term "Equilibrium Point" of a nonlinear system. **CO1** **[2]**

   d) A nonlinear system is expressed as follows: **CO4** **[12]**

$$\dot{x}_1 = -x_1 + x_2(1 + x_1)$$
$$\dot{x}_2 = -x_1(1 + x_1)$$

   i. Determine the equilibrium point(s) of the above system.

   ii. Linearize the above system about its equilibrium point(s).

   iii. Comment about the asymptotic stability of the system at $\boldsymbol{x}$=0.

2. a) State Lyapunov's 2nd theorem. CO1 [5]
b) The dynamics of an unforced nonlinear system is described by CO4 [10]
$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - x_1\left(x_1^2 + x_2^2\right) \\ -x_1 - x_2\left(x_1^2 + x_2^2\right) \end{bmatrix}$. Using the function $V = \frac{1}{2}\left(x_2^2 + x_1^2\right)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

OR

3. a) Define the term "phase portrait" for a nonlinear system. CO1 [5]
b) A satellite attitude control system has forward-reverse type of thrusters with a dead zone and a proportional plus derivative controller. With the help of a phase plane plot, investigate the stability of the system assuming standard notations for the parameters and variables. CO4 [10]

4. a) With schematic diagrams explain how an on-off type temperature control system functions. State the necessary controller characteristics. CO2 [5]
b) Develop expressions for the time response of the above system considering a first order plant with finite delay and an on-off type controller. CO3 [5]
c) Derive approximate expressions for on-time, off-time, duty cycle and maximum temperature. CO3 [5]

OR

5. a) A nonlinear block exhibits unity gain when the input is within +/- 5 V and saturation outside this zone. If sinusoidal excitation of +/- 10V pp is applied to the system, obtain the algebraic expression for the output waveform. CO3 [5]
b) Sketch the output waveform. CO2 [5]
c) Develop an expression for the describing function of the above nonlinearity. CO3 [5]

Part II

Answer any one question from Question nos. 6 and 7
AND
Any one question from Question nos. 8 and 9
AND
Question no. 10 which is compulsory

6. a) What is an observer? Draw the structure of a full-order observer and explain its uses. (CO1) [4]

b) Design a reduced order observer for observing the second state variable for the following continuous time system so that the observer pole is located at - 4, (CO5) [12]

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}u;\ y = [1,0]\mathbf{x}.$$

OR

7 a) Explain the physical significance of the term 'quadratic performance index'. (CO1) [2]

b) With the help of an example, explain what is meant by "minimum energy control problem". (CO1) [2]

c) Design an optimal controller for the following linear quadratic regulator by finding (CO5) [12]

(i) the Ricatti matrix P, and

(ii) the optimal control law.

The plant is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u;\ y = \begin{bmatrix} 1 & 0 \end{bmatrix}x$$

and has the performance index

$$J = \int_0^\infty \left[x^T \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x + u^2 \right] dt.$$

8. a) Distinguish between the terms 'Structured uncertainty' and 'Unstructured uncertainty. (CO2) [4]

b) Analyze the robust stability of the system with the characteristic polynomial (CO4) [12]

$p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0$,

where

$p_5 \in [2,4]$, $p_4 \in [8,12]$, $p_3 \in [5,11]$, $p_2 \in [3,6]$, $p_1 \in [4,7]$ and $p_0 \in [1,5]$.

OR

9. a) Distinguish between the terms 'Stability Robustness' and 'Performance Robustness'. (CO2) [4]

b) A process plant given by $G_1(s) = \frac{100}{(s+1)(0.01s+1)}$ is modeled by using the transfer function $G_2(s) = \frac{100}{s+1}$. (CO4) [12]

Analyze the suitability of using the above model for the given process plant on the basis of (i) the open loop unit step responses, (ii) the frequency responses of the plant and its model.

10 a) Develop the concept of system sensitivity S_α^M for a unity feedback system with forward path transfer function given by $\frac{1}{s+\alpha}$, where $M(s)$ is the closed loop transfer function of the system. CO3 [6]

b) Given a transfer function CO4 [6]

$$G(s) = \frac{12}{(s+1)(s+2)(s+3)}.$$

Determine $\|G\|_2$.

c) For the system with transfer function $G(s) = \frac{0.5s+1}{0.2s+1}$, determine $\|G\|_\infty$. CO4 [6]