Ref No.: EX/EE/ES/H/T/325/2023(S)

B .E. ELECTRICAL ENGINEERING
THIRD YEAR SECOND SEMESTER SUPPLEMENTARY EXAMINATION 2023

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS (HONS.)

Part-I

Time: Three hours | **Full Marks 100**
(√50 marks for each part)

Use a separate Answer-Script for each part

Answer any *THREE* questions

Two marks reserved for neatness and well organized answers.

Q1a) A company uses three different assembly lines—A1, A2, and A3—to manufacture a particular component. Of those manufactured by line A1, 5% need rework to remedy a defect, whereas 8% of A2's components need rework and 10% of A3's need rework. Suppose that 50% of all components are produced by line A1, 30% are produced by line A2, and 20% come from line A3. If a randomly selected component needs rework, what is the probability that it came from line A1? From line A2? From line A3?

8

Q1b) Enlist the properties of probability distribution functions (pdf) of random variables.
How can you distinguish the pdfs of discrete random variables from those of continuous random variables?
How can you derive the probability density function from the probability distribution function? Explain.

8

Q2a) The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 40 V and standard deviation 1.5 V. (i). What is the probability that the breakdown voltage of a single diode is between 39 V and 42 V? (ii) What value is such that only 15% of all diodes have breakdown voltages exceeding that value?

Use the attached table for standard Gaussian random variable. Perform linear interpolation if necessary. **8**

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Q2b) Prove the 'Lack-of Memory' property of exponential random variables. Explain the significance of this property. **8**

Q3a) Derive Markov's inequality and Chebyshev's inequality. Explain the significances of these bounds. Which one is a tighter bound and why? **8**

Q3b) Two random variables, X and Y, have a joint probability density function given by

$f(x,y) = Kxy$ for $0 \le x \le 1$; $0 \le y \le 1$

$= 0$ elsewhere.

(i) Determine the value of K that makes this a valid probability density function.

(ii) Determine the joint probability distribution function ***F(x,y)***.

(iii) Find the joint probability of the event $X \le ½$ and $Y > ½$.

(iv) Find the marginal density function *f(x)*. **8**

Q4a) If X(t) is a real wide-sense stationary (WSS) random process, examine whether or not the following are valid expressions for power spectral density. Justify your answers.

(i) $S_X(\omega) = \dfrac{6}{6+7\omega^3}$

(ii) $S_X(\omega) = \dfrac{40}{4+\omega^2}$

6

Q4b) Justify or correct the following statement with suitable reasoning/ derivation. "Magnitude of cross-correlation of two jointly WSS random processes can never exceed the product of the rms values of the processes". **6**

Q4c) What is an Ergodic Process? **4**

Q5) Write short notes on any two **8+8**

i) Gamma distribution.

ii) Probability generating function of random variables and its applications.

iii) Properties of autocorrelation functions of WSS random processes

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Table Standard normal distribution **F(z) for z= 0.00– 2.99**

Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

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B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER SUPPLEMENTARY EXAM - 2023

SUBJECT: INTRODUCTION TO STATISTICAL & PROBABILISTIC METHODS(HONS.)

Time: Three Hours

Full Marks: 100
(50 Marks for each part)

Use a separate Answer-Script for each part
Two marks reserved for neat and well-organized answers

Question No.	Part-II	Marks

Answer any three questions

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	1.6	1.65	1.70	1.80	1.90	1.95	2.0	2.5
P (z):	0.9452	0.9505	0.9554	0.9641	0.9713	0.9744	0.9772	0.9938

z:	2.55	2.60	2.65	2.70
P (z):	0.9948	0.9953	0.9960	0.9965

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1. (a) Establish the relations between the expectation and the variance of the sample mean and the corresponding quantities of the population. Also discuss the significance of the central limit theorem. **8**

   (b) Civil engineers believe that W, the amount of weight (in units of1,000 pounds) that a certain span of a bridge can withstand without structural damageresulting, is normally distributed with mean 400 and standard deviation 40. Supposethat the weight (again, in units of 1,000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. How many cars would have to be on the bridge span for theprobability of structural damage to exceed 0.1? **8**

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2. (a) Explain the significance of the term '95% upper confidence interval' in connection with estimation of the mean of a normal population with known variance. 8

(b) A signal having value μ is transmitted from location A and the value received at B is normally distributed with mean μ and variance 6.25. If the successive received values are

6.5, 8.0, 14.5, 9.5, 10.5, 5.0, 7.5, 12.5 & 8.5,

then determine the 95% confidence interval for μ. Also determine its 99% upper confidence interval. 8

3. (a) Define the terms, (i) Simple Hypothesis, (ii) Composite Hypothesis, (iii) Type I error, (iv) Type II error, (v) Critical Region. 10

(b) Suppose it is known that if a signal of value μ is sent from station A, then the value received at station B is normally distributed with mean μ and variance 4. It is suspected that the signal value μ=8 will be sent today from A. Test this hypothesis if the same signal value is independently sent five times and the average value received at B is $\overline{X}$=9.5. Consider 5% significance level. 6

4. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. 8

(b) Determine the expected values of Regression Parameters of a simple linear regression model. 8

5. (a) What is "continuity correction"? Explain it with a suitable example. 6

(b) If $\overline{X}$ is the sample mean and S^2 is the sample variance from a normal population with mean μ and standard deviation σ, then show that $(n-1)S^2/\sigma^2$ has a chi-square distribution with (n-1) degrees of freedom and $\sqrt{n}(\underline{X} - \mu)/S$ has a t- distribution with (n-1) degrees of freedom where n is the sample size. 10