B .E. ELECTRICAL ENGINEERING
THIRD YEAR SECOND SEMESTER EXAMINATION 2023

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS (HONS.)

Time: Three hours **Full Marks 100**
(√50 marks for each part)

Use a separate Answer-Script for each part

Q1a) State and explain the axioms of probability. (CO1)

Q1b) A company sells high fidelity amplifiers capable of generating 10, 25 and 50W of audio power. It has on hand 100 of the 10W units, of which 15% are defective, 70 of the 25W units with 10% defective and 30 of the 50W units with 10% defective.

i) What is the probability that an amplifier sold from the 10W units is defective?
ii) If each wattage amplifier sells with equal likelihood, what is the probability of a randomly selected unit being 50W and defective?
iii) What is the probability that a unit randomly selected for sale is defective?
(CO1) 2+6

Q2a) A production line manufactures 1000 Ohm resistors that must satisfy a 10% tolerance.

i) If resistance is adequately described by a Gaussian random variable X for which μ_X =1000 Ohm and σ_X = 40 Ohm, what fraction of the resistors is expected to be rejected?
ii) If a machine is not properly adjusted, the product resistances change to the case where μ_X =1050 Ohm (5% shift). What fraction is now rejected?

Use the attached table for standard Gaussian random variable.
(CO2) 6

Q2b) Starting from the expression for the exponential probability density function, show that the sum of several independent and identically distributed exponential random variables is a Gamma random variable.
(CO2) 4

Q2c) Use Chebyshev's theorem to find what percentage of the values will fall between 123 and 179 for a dataset with mean of 151 and standard deviation of 14.
(CO2) 4

[Turn over

OR

Q2a) Prove that Poisson distribution can be used as a convenient approximation to the Binomial distribution under some assumptions. (CO2) 6

Q2b) A noisy transmission channel has a per digit error probability p=0.01.
i) Calculate the probability of more than one error in 10 received digits.
ii) Calculate the same probability using Poisson approximation.
(CO2) 4

Q2c) A random variable has a probability density

$$f_X(x) = (5/4)(1-x^4) \quad 0 < x \leq 1 \; and \; 0 \; elsewhere$$

Find E[X], E[4X+2] and E[X^2].
(CO2) 4

Q3a) Two random variables X and Y have a joint probability density function (CO3) 6

$$f_{X,Y}(x,y) = \frac{5}{16} x^2 y \quad 0 < y < x < 2 \; and \; 0 \; elsewhere$$

i) Find the marginal density functions of X and Y.
ii) Are X and Y statistically independent?

Q3b) Find the value of the constant b in terms of a so that the following function becomes a valid joint density function. (CO3) 3

$$f_{X,Y}(x,y) = be^{-(x+y)} \quad 0 < x < a, 0 < y < \infty \; and \; 0 \; elsewhere$$

Q3c) Introduce the concept of joint conditional probability distribution and density functions. (CO3) 5

OR

Q3a) Let Y=$aX+b$ where X and Y are two random variables, a and b are constants and a can assume both positive and negative values. (CO3) 6

i) Find the covariance of X and Y.
ii) Find the correlation coefficient of X and Y.
iii) Comment on the correlation between X and Y.

Q3b) Prove that the variance of a weighted sum of uncorrelated random variables (weights α_i) equals the weighted sum of the variances of the random variables. (CO3) 4

Q3c) Let Y = aX+ b. If $\Psi_X(\omega)$ is the characteristic function of X, then determine the characteristic function of Y. (CO3) 4

Q4) Write short notes on any two (CO4) 7+7

i) Stationarity and Ergodicity
ii) Cross-Correlation function and its properties
iii) Power Spectral Density of a Random Process

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Table Standard normal distribution **F(z) for z= 0.00– 2.99**

Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

for $z > 2.99$ assume $F(z) = 0.9999$

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B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER - 2023

SUBJECT: INTRODUCTION TO STATISTICAL & PROBABILISTIC METHODS(HONS.)

Time: Three Hours | Full Marks: 100
(50 Marks for each part)

Use a separate Answer-Script for each part

Two marks for neat and well-organized answers

Question No.	Part-II	Marks

Answer any three questions

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	1.6	1.65	1.70	1.80	1.90	1.95	2.0	2.5
P (z):	0.9452	0.9505	0.9554	0.9641	0.9713	0.9744	0.9772	0.9938

z:	2.55	2.60	2.65	2.70
P (z):	0.9948	0.9953	0.9960	0.9965

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1. (a) If $\overline{X}$ is the sample mean and S^2 is the sample variance from a normal population with mean μ and standard deviation σ then show that $(n-1) S^2/\sigma^2$ has a chi-square distribution with (n-1) degrees of freedom. **8**

(b) An astronomer wants to measure the distance from her observatory to a distant star. However, due to atmospheric disturbances, any measurement will not yield the exact distance *d*. As a result, the astronomer has decided to make a series of measurements and then use their average value as an estimate of the actual distance. If the astronomer believes that the values of the successive measurements are independent random variables with a mean of *d* light years and a standard deviation of 2 light years, how many measurements need she make to be at least 95 percent certain that her estimate is accurate to within ±.5 light years? **8**

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2. (a) Derive the confidence interval for estimation of the mean of a normal population with known variance. 8

(b) A signal having value μ is transmitted from location A and the value received at B is normally distributed with mean μ and variance 4. If the successive received values are 8

5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5

then determine the 95% confidence interval for μ. Also determine its 99% upper confidence interval.

3. (a) Derive the expression of sample size requirement to meet certain specifications concerning type-II error. 10

(b) Suppose it is known that if a signal of value μ is sent from station A, then the value received at station B is normally distributed with mean μ and variance 4. It is suspected that the signal value μ=8 will be sent today from A. Test this hypothesis if the same signal value is independently sent five times and the average value received at B is $\overline{X}$ =9.5. Consider 5% significance level. 6

4. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. 8

(b) Determine the expected values of Regression Parameters of a simple linear regression model. 8

5. (a) What is "continuity correction"? Explain it with a suitable example. 6

(b) The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college. 6

(c) Derive the expression of variance of n number of samples. 4