Ref No: Ex/CSE/PC/B/T/225/2023

Bachelor in Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Exam 2023
Graph Theory and Combinatorics

Full Marks : 100 Time : 3 Hrs

Write answers to the point. Make and state all the assumptions(wherever made).
ALL PARTS OF A QUESTION SHOULD BE ANSWERED TOGETHER

Section A Answer all questions [$7 \times 5 = 35$]

(1) How many different strings can be made by reordering the letters of the word "MISSISSIPPI "?

(2) What is the generating function for the sequence 1, 1, 1, 1, 1, 1?

(3) A small merry-go-round has 8 seats occupied by 8 children. In how many ways can the children change places so that no child sits behind the same child as on the first ride? The seats do not matter, only the relative positions of the children.

(4) What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$?

(5) Prove that in finite graph, the number of vertices of odd degree is always even.

(6) For each of the following, draw an Eulerian graph that satisfies the conditions, or prove that no such graph exists.

(a) An even number of vertices, an even number of edges.
(b) An even number of vertices, an odd number of edges.
(c) An odd number of vertices, an even number of edges.
(d) An odd number of vertices, an odd number of edges.

(7) Can five houses be connected to two utilities without any connections crossing each other. Justify.

Section B Answer any Five(5) Questions. [$5 \times 5 = 25$]

You can keep your answer in the "Combinatorial Form"

(1) Find the number of solutions to $x_1 + x_2 + x_3 + x_4 = 20$ with $x_1 \geq 0, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1$.

(2) What is the closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

(3) In a shop there are k kinds of postcards. We want to send postcards to n friends. How many different ways can this be done? What happens if we want to send them different cards? What happens if we want to send two different cards to each of them (but different persons may get the same card) ? (5)

(4) Show that if n pigeonholes shelter $kn+1$ pigeons, where k is a positive integer, at least 1 pigeonhole shelters at least $k + 1$ pigeons

(5) Prove by induction that every third element in a Fibonacci sequence is an even number.

(6) There are 4 roads from A and B and 6 roads from B and C. Show that the number of ways to go

(a) from A to C is 24
(b) from A to C and back to A is 576

[Turn over

Ref No: Ex/CSE/PC/B/T/225/2023

(c) from A to C and back to A without using a road more than once is 360

Section C Answer any Five(5) Questions [$5 \times 5 = 25$]

(1) Prove or disprove (i) K_5 is non-planar (ii) $K_{3,3}$ is planar.

(2) Show that there is no simple graph with 12 vertices and 28 edges in which (i) the degree of each vertex is either 3 or 4 and (ii) the degree of each vertex is either 3 or 6

(3) Prove or disprove: No digraph contains an odd number of vertices of odd outdegree or an odd number of vertices of odd indegree.

(4) Show that the Petersen graph does not have a Hamilton circuit, but the subgraph obtained by deleting a vertex v has a Hamilton circuit.

(5) Obtain an Eulerian circuit of the graph shown in Figure 1

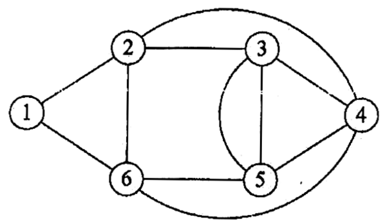

Figure 1:

(6) Obtain a path between vertices v_1 and v_3 from the adjacency matrix of the graph $G(V, E)$ in Figure 2

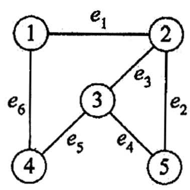

Figure 2:

Section D Answer any one question [$8 + 7 = 15$]

(1) (a) In chess, a rook attacks any piece in the same row or column as the rook, provided no other piece is between them. In how many ways can eight indistinguishable rooks be placed on a chess board so that no two attack each other? What about eight indistinguishable rooks on a 10×10 board?

(b) Using techniques from graph theory, show that $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

(2) (a) Show it is not possible tohave a group of seven people such that each person in the group knows three other people in the group

(b) Prove combinatorially that

$$C(3n, 3) = 3C(n, 3) + 6nC(n, 2) + n^3$$