5. a) If the matrix A satisfies $A^2 - A + I = 0$, where I is the unit matrix, then prove that A^{-1} exists and is equal to (I – A).

   b) Solve the equations by matrix method :

$$x + 2y + 3z = 14$$
$$2x - y + 5z = 15$$
$$-3x + 2y + 4z = 13.$$

4+6

6. a) What is the length of the subtangent and subnormal of a curve $y = f(x)$ at any point?

   b) Show the the subnormal for the parabola $y^2 = 4ax$ is constant.

   c) If the two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ cuts orthogonally, then show that $\frac{1}{b} - \frac{1}{b'} = \frac{1}{a} - \frac{1}{a'}$.

2+3+5

7. a) What is the radius of curvature of a curve $y = f(x)$?

   b) Find the radius of curvature at any point of the curve $x = a(\Theta + \sin\Theta)$, $y = a(1 - \cos\Theta)$.

   c) If the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ makes an angle ϕ with the x-axis, then show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$. 2+4+4

Bachelor of Architecture Examination, 2023

(1st Year, 2nd Semester)

Mathematics-II

Time : Three hours Full Marks : 100

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (50 Marks)

Answer ***any five*** questions.

1. a) If the points A and B are (2, 3, –6) and (3, –4, 5), find the direction cosine of the line AB. 5

   b) Find the direction cosine of the line which is equally inclined to the axes. 5

2. a) Find the equation of the plane passing through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and perpendicular to the plane $2y - 3z = 4$. 6

   b) A plane meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$. 4

3. a) Find the equation of the line through the point (1, 2, –1) and perpendicular to each of the lines

$$\frac{x}{1}=\frac{y}{0}=\frac{z}{-1} \text{ and } \frac{x}{3}=\frac{y}{4}=\frac{z}{5}.$$ 5

b) Find the equation of the line $x+y+z-1=0$, $2x-y-3z+1=0$ in symmetrical form. 5

4. Prove that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of their point of intersection. 10

5. Find the equation of the sphere for which the circle $x^2+y^2+z^2+2x-4y+5=0$, $x-2y+3z+1=0$ is a great circle. 10

6. Find the magnitude and the equation of the line of shortest distance between the lines

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text{ and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z-1}{1}.$$ 10

7. a) Find the equation of the sphere passing through the points (0, 0, 0), (–1, 2, 0), (0, 1, –1) and (1, 2, 5). 4

b) Show that the general equation of a cone of 2nd degree which passes through the coordinate axes is of the form $fyz+gzx+hxy=0$. 6

Part – II (50 Marks)

Answer ***any five*** questions.

1. a) Define adjoint (Δ') of a determinant (Δ) and hence show that $\Delta' = \Delta^2$.

b) Show that the adjoint of a symmetric determinant is symmetric. 5+5

2. Solve by Cramer's rule :

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$3x+2y+9z=14.$$ 10

3. a) Define inverse of a square matrix of order n and hence show that the inverse of the matrix, if exists, is unique.

b) If A and B be two non-singular square matrices of the same order, then show that the inverse of product of A and B is the product of their inverses in the reverse order. 4+6

4. a) Define an orthogonal matrix.

b) Show that the value of the determinant of an orthogonal matrix is ± 1.

c) Prove that every square matrix can be expressed as a sum of a symmetric matrix and a skew-symmetric matrix uniquely. 2+4+4

[Turn over