BACHELOR OF ARCHITECTURE EXAMINATION, 2023

(1st Year, 1st Semester)

MATHEMATICS – I

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use separate Answer-Script for each part.

Part – I (50 Marks)

Answer question **No. 1** and ***any three*** from the rest.

1. If $y = \sin^{-1} x$, then prove that

   i) $(1-x^2)y_2 - xy_1 = 0$.

   ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. 5

2. a) Verify Rolle's theorem for the function $f(x) = \ln(x^2+2) - \ln 3$ in [–1, 1]. 7

   b) State Cauchy's Mean value theorem and hence deduce the Lagrange's Mean value theorem. 8

3. a) Find the value of θ, in the Mean value Theorem

   $f(a+h) = f(a) + hf'(a+\theta h)$, where $f(x) = \sqrt{x}$, $a = 1$ and $h = 3$. 8

   b) Apply Maclaurin's theorem to $f(x) = (1+x)^4$ to deduce that

   $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$ 7

4. a) If $f'(x)$ exists at $x = a$, then prove that

   $$\lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h} = f'(a)$$ 9

   b) Evaluate : $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$. 6

5. a) If $u = f(x^2 + 2yz, y^2 + 2zx)$, then show that

   $$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$$ 8

   b) Show that

   $$\frac{\pi^2}{9} \le \int_{\pi/6}^{\pi/2} \frac{x}{\sin x}\,dx \le \frac{2\pi^2}{9}.$$ 7

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Ref. No.: Ex/Arch/Math/T/114/2023

BACHELOR OF ENGINEERING IN ARCHITECTURE ENGINEERING EXAMINATION, 2023

(1st Year, 1st Semester)

Mathematics-I

Time: Three hours | Full Marks: 100

(50 marks for each Part)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each Part

PART-II (50 Marks)

Answer ***Q. No.1*** and any ***three*** from the rest

1. Find the whole length of the curve $x^2(a^2 - x^2) = 8a^2y^2$. **5**

2. Examine the convergence of the following improper integrals:
   a) $\int_0^1 \frac{x^{p-1}}{1+x} dx$, (b) $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}} dx$, (c) $\int_0^\infty x^{-\frac{1}{2}} e^{-x} dx$. **15**

3. a) Define Beta and Gamma function.

   b) Show that $B(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta\, cos^{2n-1}\theta\, d\theta, m > 0, n > 0$.

   c) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx$.

   d) Changing the order of integration, evaluate $\int_0^\infty \int_0^\infty e^{-xy} \sin nx\, dxdy$.
   Hence deduce that $\int_0^\infty \frac{\sin nx}{x} \mathrm{dx} = \frac{\pi}{2}$. **3+3+2+7**

4. a) State the fundamental theorem of integral calculus.

   b) A function f is defined on $[-2, 2]$ by $f(x) = \begin{cases} 3x^2 \cos\frac{\pi}{x^2} + 2\pi \sin\frac{\pi}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

   Show that f is Riemann integrable on $[-2, 2]$ and then evaluate $\int_{-2}^{2} f\, dx$.

   c) Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$, the field of integration being the positive octant of the sphere $x^2 + y^2 + z^2 = 1$. **2+5+ 8**

5. a) Calculate the value of $\int_0^1 \sqrt{1 - x^3}\, dx$ using Simpson's $\frac{1}{3}$ rule by taking six intervals.

   b) Find the volume of the solid obtained by revolving the cardioide $r = a(1 + \cos\theta)$ about x-axis. **7+8**