Bachelor of Architecture Examination, 2023

(1st Year, 1st Semester, Supplementary)

Mathematics - I

Time : Three hours | Full Marks : 100

Use separate Answer script for each Part.

50 marks for each Part.

Symbols / Notations have their usual meanings.

Part – I (50 Marks)

Answer question **No. 1** and ***any three*** from the rest.

1. If $y=\sin\left(m\sin^{-1}x\right)$, show that

$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2-m^2\right)y_n=0$. 5

2. a) Verify the Rolle's theorem for the function

$f(x)=x(x+3)e^{-\frac{x}{2}}$ in $[-3,\ 0]$. 7

b) Use mean value theorem to prove that $\sqrt{101}$ lies between 10 and 10·05. 8

3. a) Prove that

$l\text{n}(1+x)>x-\dfrac{x^2}{2}$, if $x>0$. 6

b) If $f''(x)$ exists at $x=a$, then prove that

$\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}=f''(a)$. 9

4. a) Expand $5x^2+7x+3$ in powers of $(x-3)$. 8

b) Evaluate: $\lim_{x\to 0}\frac{e^x - e^{\sin x}}{x-\sin x}$. 7

5. If $v=\log\left(x^3+y^3+z^3-3xyz\right)$, then show that

a) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)v=\frac{3}{x+y+z}$. 7

b) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v=-\frac{3}{(x+y+z)^2}$. 8

Part – II (50 Marks)

Answer question **No. 1** and ***any three*** from the rest.

Answer the following questions:

1. Evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\cot x}\, dx$. 5

2. Examine the convergence of the following integrals:

a) $\int_2^{\infty}\frac{dx}{\log x}$ b) $\int_0^{\infty}\frac{\sin x}{x}dx$ c) $\int_0^2\frac{1}{\sqrt{x(2-x)}}dx$ 15

3. a) Find the surface area of the solid obtained by revolving one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about x-axis.

b) Using a double integral, prove that $B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$; $a,b>0$.

c) Evaluate $\int_0^{\infty}e^{-ax^2}dx$. $(a>0)$ 7+5+3

4. a) Calculate the value of $\int_2^4\frac{x}{x-1}dx$ using Simpson's $\frac{1}{3}$ rule by taking eight intervals.

b) Let $f:[-3,\ 3]\to R$ be defined by

$$f(x)=\begin{cases}2x\sin\frac{\pi}{x}-\pi\cos\frac{\pi}{x} & x\neq 0\\ 0 & x=0\end{cases}.$$

Examine whether f is Riemann integral in [–3, 3] and hence find $\int_{-3}^{3}f\ dx$.

c) Compute the length of one arch of the cycloid $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$. 5+5+5

5. a) Evaluate $\iint_R\frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}dxdy$, where R is the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

b) Find the area of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

c) State condition for convergence of beta function.

7+6+2