Ref. No.: SC/INSC/PG/CORE/TH/MI101/2023

M.SC. INSTRUMENTATION 1ST YEAR 1ST SEMESTER - 2023

SUBJECT: **ADVANCE MATHEMATICS AND COMPUTER PROGRAMMING**
Time: 4 Hours Full Marks: 80

Part-I

Use separate answer scripts for each part; Answer any **four** questions

1. a) State Cauchy Riemann equations. 1+9

   b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions:

   (i) u = 0 when x = 0, t > 0 (ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ $when\ t = 0$ (iii) u(x.t) is bounded.

2. (a) Find the solution of the heat conduction equation
   $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$,
   for which $u(0,t) = u(l,t) = 0$ and $u(x,0) = \sin\frac{\pi x}{l}$ by the method of separation of variables.
   (b) Using the Cauchy Riemann equations, show that $f(z) = z^3$ is analytic in the entire z plane.
   (c) Show that the function $u(x,y) = 4xy - 3x + 2$ is harmonic. 5+3+2

3. (a) State the Dirichlet's conditions.
   (b) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}$dx. 2+4+4

4. (a) What are the required conditions for the convergence of the Fourier series of $f(x)$ to $f(x)$ at any value of x. 2+8
   (b) Find the Fourier series of the function defined as
   $$f(x) = \begin{cases} x + \pi & for \quad 0 \leq x \leq \pi \\ -x - \pi & for \ -\pi \leq x < 0 \end{cases} \quad \text{and} \quad f(x + 2\pi) = f(x).$$

5. (a) Show that the Fourier transforms of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transforms, $F[f(x) * g(x)] = F[f(x)].F[g(x)]$. 4+3+3
   (b) Prove that, $L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$, where the symbols have their usual meaning.
   (c) Prove that, $L\left[\frac{1}{t} f(t)\right] = \int_s^\infty F(s) ds$, the symbols have their usual meaning.

6. a) Test the analyticity of the function $w = \sin z$.
   b) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to π and show that
   $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ 3+7

7. Express the following function in terms of unit step function and find its Laplace transform 10

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t - 1, & 1 < t < 2 \\ 1, & 2 < t \end{cases}$$

[Turn over

(Part-II)

Answer any four questions taking two from each Group. $4\times10=40$

Group-A

1. If a number be rounded to n correct significant figures, then show that the relative error is less than $\dfrac{1}{k\times10^{n-1}}$, where k is the first significant figure in the number.

 b) Find the number of significant figures in $V_T=1.5923$ when its relative error $E_r=0.1\times10^{-3}$, where V_T is the true value of the number.

2. a)

x	1	2	3	4	5
$f(x)$	4	13	34	73	136

 Considering the above data, construct the forward finite difference table. Hence find the polynomial $f(x)$ which satisfy the above data and find the value of $f(2.5)$.

 b) Show that $\Delta\cdot\nabla=\Delta-\nabla$, where symbols have their usual meanings.

3. a) Find the expression for the Lagrange's interpolation formula.

 b) Using Lagrange's interpolation formula compute $f(2)$ from the table given below:

x	0	1	3	4
$f(x)$	5	6	50	105

4. a) Solve the following system of simultaneous linear equations using Gauss-elimination method correct upto two decimal places.

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8$$

Group – B

5. Write a general program in C-Language to solve the initial value problem $\frac{dy}{dx}=-y+x^2+1$ with $y(0)=1$ at $x=2$ with Euler method.

6. Write a general program in C-Language to take two matrices $A=\begin{pmatrix}1 & 2 & 4\\ 3 & -5 & 7\\ 4 & 7 & 8\end{pmatrix}$ and $B=\begin{pmatrix}19 & -2 & 3\\ -1 & -2 & -3\\ 41 & -7 & 15\end{pmatrix}$. Calculate $A+B$ and $A-B$.

7. Write a general program in C-Language to perform the integration $\int_1^7 2x^2-5x+2\ dx$ by Simpson's 1/3 rule.

8. Write a general program in C-Language to find the root of the equation $x^3+2x^2-5=0$ between $x=0.5$ to $x=2$.[Any method can be used]