M. Sc. Physics Examination, 2023

(2nd Year, 2nd Semester)

Dynamical Systems

Paper – 302

Time : 2 hours Full Marks : 40

Use separate answer script for each group.

Group – A

Answer ***any four*** questions from group A (each carry 5 marks)

1. The following one-dimensional system undergoes bifurcation with change of parameter *r*. Identify the type of bifurcation and obtain the bifurcation diagram.

$$\frac{dx}{dt} = rx - 9x^3 .$$

2. For the following two dimensional linear systems identify the nature of the fixed points, and draw the phase trajectories.

i) $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ and

ii) $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

3. Find the fixed points of a nonlinear system described by,

$$\frac{dx}{dt} = -x + x^3 \text{ and } \frac{dy}{dt} = 2y .$$

Use linearization to classify the fixed points and show the flow lines.

4. Define a limit cycle. Sketch the flow lines for the system.

$$\frac{dr}{dt} = r\left(1 - r^2\right)\left(4 - r^2\right) \text{ and } \frac{d\theta}{dt} = 1 .$$

5. Analyze the nature of bifurcation in the two dimensional system, $\frac{dr}{dt} = \mu r - r^3$ and $\frac{d\theta}{dt} = 1$, where μ is a parameter.

6. For the following discrete map, find the cobweb diagram. Discuss the existence and nature of fixed point(s), if any.

$$x_{n+1} = \cos\left(x_n\right)$$

Group – B

Answer ***any two*** questions.

1. Consider the dynamical system given by the equations $\dot{x} = y$ and $\dot{y} = -k\left(x^2 - 1\right)y - \omega^2 x$, with $k \gg 1$. Find the approximate time period of the limit cycle. 10

2. Find the ratio of the average kinetic energy to the average potential energy of the oscillator under the action of the restoring force given by $F = -kx^3$, where *k* is a positive constant (the averages should be evaluated over a complete time period). 10

3. For the oscillator $\ddot{x} + A\dot{x} - Bx + x^3 = 0$ with $A > 0$, what kind of bifurcation occurs when the parameter *B* goes from negative to positive values. Draw appropriate figures to show the flow lines. 10